

The Mathematics Consortium



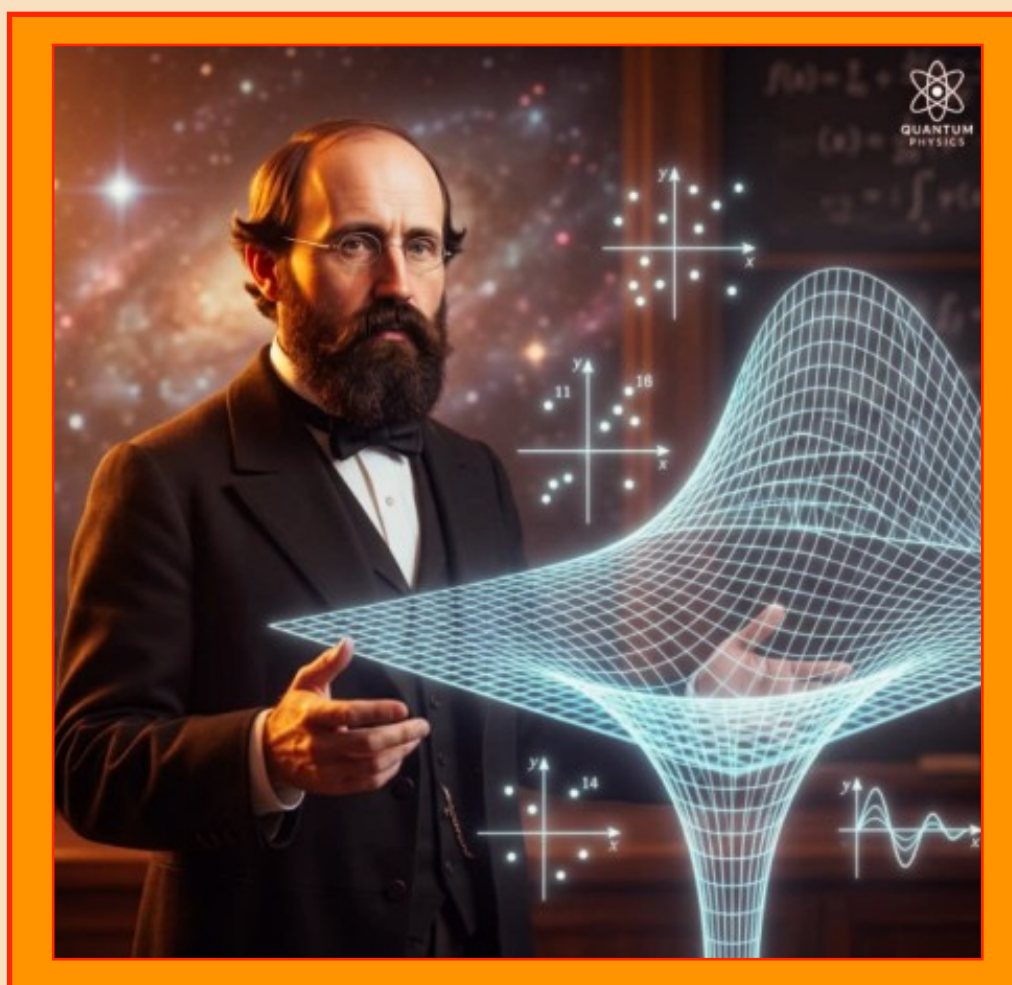
BULLETIN

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*We Pay our Tributes to the Great Mathematician and Physicist
Bernhard Riemann (1826-1866)
On the occasion of his 200 th Birth Anniversary*



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About the Cover Page: The image on the front cover is of *Bernhard Riemann*, the celebrated German mathematician, physicist and philosopher. His ideas changed the face of both mathematics and physics. His imagination reached far beyond the realm of his time. He suggested that the space need not be flat; instead it could be bent or curved. This idea later became central to Albert Einstein's work on general relativity. He is also renowned for proposing the *Riemann Hypothesis* which relates to a deep pattern hidden in the distribution of prime numbers. Another of his major achievements is the *Riemann mapping theorem*, a result in Complex Analysis which states that any simple region on a plane can be transformed into a disc through a special function.

Acknowledgement: We acknowledge with thanks that the image on the front cover is taken from Probal Chakraborty's post on Facebook, dated 11 December 2025.

From the Editors' Desk

This year, we celebrate the 200th birth anniversary of legendary mathematician, physicist and philosopher Bernhard Riemann. Despite his unfortunate early demise, before attaining 40 years, he has made everlasting impact on these three disciplines, and left so much for future generations to ponder and work on. The results he proved, and the interpretations of the physical realities he gave, have laid strong foundations for the many a development in the subsequent period. Moreover, the depth of the hypothesis he proposed is so much that even after so many years leading mathematicians have not been able to cover ample ground towards unveiling the mystery in it.

The biographies of such great personalities enlighten us in many ways. There are three main virtues that one can identify making Riemann a colossal success. One is his curiosity and keen desire to fulfil it by acquiring knowledge from the available resources. Second, a tremendous observational power and capability to put realities in the scientific framework. And the third is, comprehensive thinking to understand the hidden principles, coupled with courage to set out ideas well beyond the existing boundaries of knowledge.

A word on the educational system during Riemann's era may not be out of place here. Some distinct features were, permission to join a secondary school without formal schooling before, a teacher's freedom to exempt a student from attending classes to carry-on in-depth study of a subject individually. Even after getting Ph.D. degree, in order to teach and examine students at university, one had to acquire the Habilitation degree, by writing a dissertation and delivering a Habilitation lecture. These features provide an opportunity to a bright student to excel in the subject of his own choice.

In the opening article, (the only feature article in this Issue) Prof. Athanase Papadopoulos highlights the historical continuity in Mathematics by elaborating how Riemann was inspired by his predecessors Euclid, Legendre, Abel, Lagrange, Archimedes, Galileo and Newton, and how mathematicians Felix Klein, Hermann Weyl, André Weil, Grothendieck and others carried his legacy forward. He also, explains the influence of Gauss and Dirichlet on Riemann's work and in shaping his career. He also discusses the profound and indelible impact Riemann's work had on various branches of mathematics and Physics.

In our regular columns, in Article 2 Prof. S. G. Dani gives brief reviews of two papers on history of mathematics, one on the development of equations in ancient China, and another on the *Turagagati* method for 4×4 pandiagonal magic squares by Nārāyaṇa Paṇḍita.

In Article 3, Dr. D. V. Shah presents an account of significant developments in the world of mathematics. He highlights how high-capacity neural operators and automated formal proof engines like Lean 4 have joined forces with human analysts to tackle some of the most formidable open problems which includes: Constructing a solution of the three-dimensional incompressible Navier-Stokes equations that blows up in finite time (one of the seven Millennium Prize Problems); Verification of over 130 pages long proof of Fermat's Last Theorem given by Andrew Wiles in 1994; 50-Years-Old Cycle Double Cover Conjecture and Constructing a counter example disproving a 90-years-old Jacobian conjecture for 3 or more variables. He also gives information on the winners of 2026 Field Medals and the 2026 IMU Abacus Medal. Tributes are also paid to Prof. Nina Nikolaevna Uraltseva, who passed away recently.

In Article 4, the Problem Corner, Dr. Udayan Prajapati presents solutions to the problems from Number theory and Analysis posed in the July 2026 issue, and poses two new problems, also from Number theory and Analysis, for our readers. Dr. Ramesh Kasilingam gives a calendar of academic events, planned during December, 2026 to March, 2027, in Article 5.

Information regarding the medals and prizes won by the Indian teams participating in IMO-2026 at Shanghai, China, and IMC-2026 at Blagoevgrad, Bulgaria, is given on the back-inner cover-page.

We are happy to bring out this second issue of Volume 8 in October, 2026. We thank all the authors, all the editors, and our designers Mrs. Prajкта Holkar and Dr. R. D. Holkar, and all those who have directly or indirectly helped us in bringing out this issue on time.

Chief Editor, TMC Bulletin

1. Bernhard Riemann: Reflections on a Bicentennial

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*After a quiet preparation Riemann came forward like a bright meteor,
only to be extinguished soon afterwards.*

(Felix Klein, Development of mathematics in the 19th century, 1926,
English transl. R. Hermann)

1.1 INTRODUCTION: HISTORICAL CONTINUITY IN MATHEMATICS

This year, we celebrate the 200th anniversary of the birth of Bernhard Riemann (1826-1866). I would like to start my commemoration by saying a few general words about our mathematical ancestors, who bequeathed to us the theories we are building upon, the problems we are working on, and the tools to solve them.

More so than in the other sciences, historical continuity is particularly evident in mathematics. Every mathematician contributes, to the best of his ability, to explore this coherent and hermetic universe of mathematics — hermetic because only those who work within it can contemplate it and feel its secrets. In mathematics, and unlike in the other sciences, fundamental concepts and theories never die; they are passed down from generation to generation, and it often happens that each receiving generation can use them with almost no change. One need only think of Euclid's *Elements*, where the axioms, definitions, and propositions are still in use about 2300 years after they were written, with the very same words. Euclid is not an isolated case: Nearly two-thousand-year-old treatises by Archimedes, Apollonius, Diophantus, Theodosius, Menelaus and others could serve as bases for courses taught to our students today — and sometimes they do; the author of these lines gave courses on non-Euclidean geometry based on Menelaus's *Spherics* [27], a work written in the first century of our era. The editor of an English modern version of Leonhard Euler's *Introductio in analysin infinitorum* (Introduction to the analysis of the infinite) writes that André Weil “was trying to convince the mathematical community that our students of mathematics would profit much more from a study of Euler's *Introductio in Analysin Infinitorum*, rather than of the available modern textbooks” (see the Introduction to Vol. 1 of the English translation [7]). Mathematics, especially that written by the masters, never goes out of date.

Thus, it is not surprising that Riemann, in his writings, very frequently expresses his debt to his predecessors. For every topic on which he worked, he was thoroughly familiar with the existing literature. His 1854 Habilitation concise¹ address, *Über die Hypothesen, welche der Geometrie zu Grunde liegen* (On the hypotheses which lie at the foundations of geometry), is full of references to his predecessors: Euclid, Legendre, Abel, Lagrange, Pfaff and others. In the final section of this address, we can read:

The progress of recent centuries in the knowledge of mechanics depends almost entirely on the exactness of the construction which has become possible through the invention of infinitesimal calculus, and through the simple principles discovered by Archimedes, Galileo, and Newton, and used by modern physics.²

The references are eloquent: Archimedes, who developed the first integral calculus, allowing him to compute length, area and volume, Galileo, who introduced the modern notions of motion, velocity and acceleration, and Newton, who was the first to give a mathematical expression to the forces of nature, which allowed him to describe the motion of bodies in resisting media, and to whom is attributed a celebrated notion of space, the “Newtonian space”.

¹The original publication is 18 pages long, with generous margins.

²Regarding Riemann's Habilitation, I am using the English translation by the mathematician and philosopher W. K. Clifford (1845-1879), see [35].

As a matter of fact, the subject of Riemann's Habilitation lecture includes the three domains of Newton's major *opus*, the *Philosophiæ Naturalis Principia Mathematica* (*Mathematical Principles of Natural Philosophy*): mathematics, physics and philosophy. It is also interesting to note that Archimedes, Galileo and Newton are quoted as the three founders of mechanics in the *Discours préliminaire* (which plays the role of an Introduction) of Joseph Fourier's *Théorie analytique de la chaleur* (Analytic theory of heat) [9, p. i-ii], a work in which the author lays down the foundations of the theory of trigonometric series. Fourier's dissertation was written 32 years before Riemann's Habilitation thesis carrying the title *Über die Darstellbarkeit einer Function durch eine trigonometrische Reihe* (On the representability of a function by a trigonometric series) [34], which is precisely concerned with trigonometric series. Again, in this work, the care with which Riemann analyses the evolution of the field he is expanding, since its birth, and the wealth of historical details he gives, are another indication of how much he valued the history of mathematical ideas, and how much he was aware of the historical developments of the topics he was working on. Like in his other writings, Riemann reviews in this dissertation the important paths and sometimes warns against the wrong tracks that some of his predecessors took for the solutions of the various problems he was tackling. In the same work, he lays down the foundations of the modern theory of integration, after reviewing results and conjectures by Euler, d'Alembert, Lagrange, Daniel Bernoulli, Dirichlet, Fourier and others on trigonometric series, a topic which served as a motivation for his development of what became known later as the Riemann integral. It is also worth noting, since Riemann quotes Archimedes on several occasions, that Bourbaki considers the latter to be one of the main founders of integration theory. Let me quote Bourbaki's *Elements of History of mathematics* [4, p. 247-248]:

Concerning integration, Cauchy's work represents a return to the sound traditions of antiquity and the early 17th century, but one based on technical methods that were still inadequate. To define the definite integral, Cauchy returns to the method of exhaustion, or as we would say, to "Riemann sums" (which would be better called "Archimedes sums", or "Eudoxus sums").

One may recall here that Archimedes had developed the tools necessary for calculating areas of figures such as discs, sections of parabolas and sections of a sphere, considering analogues of what we call Riemann sums. On this matter, I would like to refer to a recent article by Demetrios Christodoulou [5]. As for Eudoxus, we owe to him the so-called *exhaustion method*, used by Euclid in Book XII of the *Elements*, a book which is due to Eudoxus. In fact, integration theory started precisely in Book XII. The exhaustion method continued developing and was greatly extended by Archimedes.

Riemann's sense of history and historical continuity is also plainly manifest in the two-page announcement of his memoir *Beiträge zur Theorie der durch die Gauss'sche Reihe $F(\alpha, \beta, \gamma, x)$ darstellbaren Functionen* (Contribution to the theory of functions representable by Gauss's series $F(\alpha, \beta, \gamma, x)$) published in 1857 (see [30] for the announcement and [31] for the memoir itself). In the announcement, like in the memoir itself [31], he presents the origin of the questions on which he was working, related to the properties and behavior of hypergeometric functions, and while doing so, he mentions the names of Wallis, Euler, Pfaff, Gauss, Gudermann, Jacobi and Kummer. There are many other examples of Riemann paying tribute to his predecessors for their work on which he built.

The next section contains, as every commemorative article should, some biographical details about Riemann. All along this section, I shall stress on his teachers, another way to honor his predecessors.

1.2 VITA AND STUDIES

As sources for Riemann's biography, we have at our disposal first-hand accounts that include letters he wrote to his family, which allow us, to some extent, to reliably trace part of his life story. The

letters are easily available, see e.g. [8], and [38] for English translations of some of them.³ Based on such documents, we can also get a sense of Riemann's feelings about his school studies, information on the courses he followed at university, his anxieties, and some of his needs. Together with notes about him written by his teachers, these documents give us some insight into his state of mind during the important period of his life when he was a school pupil, and then a university student. These letters and notes have helped various authors to write a number of biographies of Riemann.

Georg Friedrich Bernhard Riemann, usually called simply Bernhard Riemann, was born on September 17, 1826, in Breselenz, a small isolated farming and pastoral village of very few inhabitants, which was part of the Kingdom of Hanover, now in Germany. The region was poor and undeveloped. His father, Friedrich Bernhard Riemann, was a Lutheran pastor of modest means. Soon after young Bernhard's birth, the family moved to Quickborn, an even smaller village, situated about 15 kilometers from Breselenz. This sums up the environment in which the future mathematician, who was the second of six siblings, spent his childhood.

Until age fourteen, Bernhard Riemann received his teaching at home, mostly from his father, because there was no nearby school. In 1840, his parents sent him to Hanover, where he entered secondary school, living with his grandmother. The latter died two years later, and his family moved him to the Johanneum Gynnasium in Lüneburg,⁴ another town in the Kingdom of Hanover, where he lived as a boarding pupil, from 1842 to 1846. The school had a good reputation. The inscription *doctrinae, virtuti, humanitati* (To learning, to virtue, and to humanity) hung above its entrance. The focus of instruction was on the classical languages. Riemann's mother, Charlotte Ebell, died in 1846.

Initially, the young Bernhard was meant to become, like his father, a Protestant pastor. Encouraging him in this direction was a natural choice for his parents, as it would have been a way to ensure that their son would have a livelihood and a social standing. Those who knew him at that time, including his high school teachers, said that he developed a special interest in mathematics at an early age, though it was not his unique favorite subject; he enjoyed studying in general and was gifted in all areas, especially the classics.

Since his childhood, Bernhard Riemann was a lonely boy, subject to depression and with difficulties to interact with his schoolmates. He used to return home whenever he could, to be with his family; it was the only place where he felt comfortable. But those opportunities were not very frequent, since traveling was expensive and complicated.

Riemann liked history. In a letter to his father, dated May 3, 1840 (he was 14), he complains about the fact that at the Gymnasium there were fewer lessons on history than on Rechnen (computing). On August 5, 1841, he writes, again to his father, that he has the highest grade, among his classmates, in history. Besides history, Riemann was doing very well in Greek, Latin and German composition (letters of February 1, 1845 and March 8, 1845). According to another letter to his father, dated April 30, 1845, it is only at that point that he started being seriously attracted by mathematics. He mentions that his mathematics teacher, Constantin Schmalfuss, seeing his capabilities, advised him to pursue studies in mathematics.

Schmalfuss was also the Director of the Lüneburg Gymnasium. When he noticed how easily his pupil was making progress in mathematics, and the degree of maturity he had attained in a short time, he decided to exempt him from taking his class and allowed him to borrow advanced mathematics books from his own library and to read them on his own. Riemann took this opportunity and started studying these books. This is how he became familiar at an early age with

³Some dates differ from source to source and there may be mistakes in the transcription of the dates. We have done our best to transmit the content of some of these letters and to follow the natural chronology.

⁴Neuenschwander, in [8], says that Riemann, after spending two years at Hanover's high school, was sent by his parents to the Johanneum Lüneburg, following difficulties with his teachers in Hanover. To support this, the author refers to a correspondence exchanged between Riemann and his father in 1842. The same author notes that Riemann's certificate upon leaving the high school in Hanover included the remark that, in recent times, he had drawn criticism for his childish behavior (*kindisches Benehmen*).

some foundational mathematical texts, including works of great mathematicians of the past such as Euclid, Archimedes, Descartes, and Newton. On one occasion he borrowed a nine hundred page book by Legendre on number theory which, it is said, he read in six days.

In the same letter to his father, April 30, 1845, Riemann declares that he plans to enroll the University of Göttingen to study theology, but that in reality he must think more, before he decides what to do, since, if he does not take the right way, he “will bring nothing good to any subject.” The last sentence, the will of “bringing something good to a subject”, is remarkable, from a high-school pupil, and it shows how mature he was at age 19.

On April 25, 1846, Riemann enrolled at the faculty of theology of the University of Göttingen, with the encouragements of his father. While he was there, he managed to attend lectures on mathematics. In particular, during the winter semester 1846/47, he followed a course by Carl Friedrich Gauss and another one by Moritz Stern⁵ on definite integrals. For each course, he had to pay some amount of money. Universities in Europe were not tuition-free, as they are today. In a letter to his father dated November 15, 1846, Riemann writes [38]: “I have enrolled in six courses, for a total cost of three Louis d’or and three Thaler.”⁶ Namely, Cultural History of Greece and Rome, by Hermann, Introduction to the Study of Theology by Ehrenfeuchter, Recent Church History by Reincke, General Physiology by Lotze, The Definite Integral by Stern, and the Theory of least squares by Gauss.” He adds: “General natural history would be very interesting, and I would certainly attend, if along with everything else I had enough money.”

Neuenschwander, in [8], reports that the University of Göttingen has kept a list of books that Riemann borrowed from its Library. These records show that Riemann, at that time, was interested not only in Gauss’s works but also, and above all, in works of some French mathematicians. He borrowed Cauchy’s *Course in Analysis, Differential Calculus*, and *Mathematical Exercises*, and Legendre’s *Treatise on Elliptic Functions and Eulerian integrals*.⁷

After this, Riemann asked his father the permission to transfer to the faculty of philosophy so that he could study mathematics, and his father responded positively. And thus began the brief but dazzling career of the man who arguably is the most brilliant mathematician of modern times.

Between 1847 and 1849, with a break in 1848,⁸ Riemann studied mathematics at the University of Berlin. His professors included Jakob Steiner, Charles Gustave Jacob Jacobi, Johann Peter Gustav Lejeune Dirichlet and Gotthold Eisenstein, that is, some of the greatest German mathematicians of that generation.

It appears, from the thoughts Riemann noted down, that Dirichlet was the teacher he admired the most. Johann Peter Gustav Lejeune Dirichlet (1805-1859), who taught in Berlin from 1828 to 1855, is considered the founder of analytic number theory, with fundamental contributions to trigonometric series (not unrelated to number theory), and to potential theory. These are all topics that were dear to Riemann. Later, Riemann met Dirichlet again in Göttingen, where Riemann became a student and Dirichlet served as a professor. After Dirichlet passed away in 1859, Riemann succeeded him as a professor and head of the mathematics department at the University of Göttingen.

⁵Moritz Abraham Stern (1807-1894), who was teaching at Göttingen, had earned his doctorate at that same university, before a jury chaired by Gauss. A class of prime numbers is named after him.

⁶This is a mixed gold and silver coinage used in past accounting. If the transcription in [38] is correct, this means that the tuition at the University of Göttingen was very costly.

⁷This treatise by Legendre was published in three volumes between 1825 and 1828. The author establishes there a classification and conducts an in-depth study of elliptic integrals and functions, laying important groundwork for this topic and for the subsequent works of Abel, Jacobi, and Riemann on it.

⁸Major disturbances took place in Berlin in 1848, linked to a popular uprising aimed at demanding a constitution, freedom of the press, and German unity. The army fired on the crowd, triggering intense street fighting and the erection of barricades. The disturbances continued into 1849. We learn from [8] that in the spring of 1849, yielding to his father’s will, who was alarmed by the unrest in Berlin, Riemann returned to Göttingen, where the situation was calmer.

Dirichlet's lectures in Berlin, at that epoch, were centered mostly on theoretical physics, with a stress on partial differential equations. It is from these lectures that Riemann became familiar with potential theory, a topic which was to play an important role in his later work, especially in his work on uniformization. Klein, in his work *Development of mathematics in the 19th century* [12, p. 234-235], published posthumously in 1926 and 1927, writes the following:

Dirichlet loved to make things clear to himself in an intuitive substrate; along with this he would give acute, logical analyses of foundational questions and would avoid long computations as much as possible. His manner suited Riemann, who adopted it and worked according to Dirichlet's methods.



Figure 1: Johann Peter Gustav Lejeune Dirichlet (1805-1859)
had a profound influence on Riemann

Riemann's admiration for Dirichlet is expressed at several places in his writings, for instance in the third section of the historical part of his Habilitation dissertation on trigonometric series which we already mentioned. Speaking of trigonometric series, Riemann describes Dirichlet as the mathematician who laid a solid foundation for a great deal of important analytical research. He recalls that Dirichlet highlighted an error made by Euler and succeeded in clarifying a question that had occupied so many eminent geometers over the preceding seventy years. He clearly explains how his own work draws inspiration from Dirichlet's and builds upon it. He then explains how Fourier series, in addition to their applications in physics, are also used in pure mathematics — more specifically, in number theory — and that Dirichlet was a pioneer in this field as well.

During his stay at Berlin, Riemann also attended lectures on optics by the physicist Wilhelm Dove (1803-1879), which he found very interesting, and on astronomy, by the prominent German astronomer Johan Franz Enke (1791-1865) (letter without date, quoted in [38]). About the latter, Riemann says that “his presentation for the most part is rather dry and boring, however the time that we spend at the observatory once a week, from 6pm to 8pm, is useful and instructive.”

Besides Dirichlet, one may mention Jacobi. In a letter to his father, dated May 30, 1849, Riemann writes [38]: “Jacobi has just begun a series of lectures in which he leads off once again with the entire system of the theory of elliptic functions in the most advanced, but elementary way.” In another letter (written around the same period but without date), also sent from Berlin, Riemann writes: “I enrolled with five other students into a private class (*Privatissimum*) with Eisenstein, who was promoted in the course of this semester to a Privatdozent, with a paper on the theory of elliptic functions.”

In the fall of 1849, after two years spent in Berlin, Riemann returned to Göttingen, where he began working on his doctoral dissertation under Gauss's supervision. But the first contact

between Gauss and Riemann had taken place before Gauss officially became Riemann's mentor. Already in 1846, in a letter dated November 5, Riemann informing his father about the courses he plans to follow, mentions a course by Gauss on the method of least squares.⁹ After that, and during his two year stay in Berlin (1847-1849), Riemann continued studying thoroughly Gauss's papers. In a letter to his father, dated May 30, 1849, he writes:

Dirichlet has arranged to me to have access to the library. Without his assistance, I fear there would have been obstacles. I am usually in the reading room by nine in the morning, to read two papers by Gauss that are not available anywhere else. I have looked fruitlessly for a long time in the catalog of the Royal library for another work of Gauss, which won the Copenhagen prize, and finally just got it through Dr. Dale of the Observatory. I am still studying it.¹⁰

Regarding his written production, Riemann endorsed Gauss' principle: *pauca sed matura* (few but ripe).

During the same stay in Berlin, Riemann followed lectures by Dirichlet on topics related to Gauss's works. He writes to his father (letter without date): "My own course of specialization is the one with Dirichlet; he lectures on an area of mathematics to which Gauss owes his entire reputation. I have applied myself very seriously to this subject, not without success, I hope."

In Göttingen, after his return in 1849, Riemann attended lectures and seminars by the newly hired physics professor Wilhelm Eduard Weber (1804-1891), who was also Gauss's collaborator and close friend. Klein writes, in the *Development of mathematics in the 19th century* [12, p. 235]:

In Weber, Riemann found a patron and a fatherly friend. Weber recognized Riemann's genius and drew the shy student to him. [...] Riemann's interest in the mathematical treatment of nature was awakened by Weber, and Riemann was strongly influenced by Weber's questions.

Riemann's doctoral dissertation was defended at the University of Göttingen in 1851. This dissertation, titled *Grundlagen für eine allgemeine Theorie der Functionen einer veränderlichen complexen Grösse* (Foundations for a General Theory of Functions of a Complex Variable) [28], is probably the most important piece of work to appear as a doctoral thesis. It was submitted on December 16, 1851. In a letter to his brother, dated November 26, 1851, Riemann writes that after he delivered his doctoral dissertation manuscript, Gauss took it home and examined it for a few days, but that before reading it, Gauss told him that for years he had been preparing an essay, on which he is still occupied, whose subject is the same, at least in part, as that covered by this dissertation. He also told him that he had already expressed his intention to write on that subject in his doctoral dissertation which, at that time, was already 52 years old.

After Riemann obtained his doctoral degree, and upon Gauss's recommendation, he was admitted, again at Göttingen University, to work on his Habilitation, the degree which would allow him to be a Privatdozent, that is, to teach and examine students at university.¹¹ He spent thirty

⁹This is a method for minimizing the sum of the squared residuals, that is, the differences between observed values and the values predicted by the model. Gauss developed the method of least squares around 1794-1795 in his work on tracking planetary and stellar orbits.

¹⁰Gauss's paper that won the Copenhagen prize, is titled *Allgemeine Auflösung der Aufgabe: die Abbildung des Einen auf eine andere Fläche so einzurichten, dass die Abbildung dem Abgebildeten in den kleinsten Theilen ähnlich wird* (General solution of the problem: to map one surface onto another so that the image is similar to the original in its smallest parts) [10]. It was published in 1825 in the *Astronomische Abhandlungen* (Memoirs on astronomy). This paper won a prize for a competition proposed by the Copenhagen Royal Society of Sciences in 1822 whose subject was: "To represent the parts of a given surface onto another surface in such a way that the representation is similar to the original in its infinitesimal parts." In this paper, Gauss proved his famous result saying (in modern terms) that on a real-analytic surface, every sufficiently small neighborhood of an arbitrary point can be mapped conformally onto a subset of the plane. Gauss did not solve the general problem of mapping conformally an arbitrary finite portion of the surface. This question was settled by Riemann, after Gauss, in his doctoral dissertation. The so-called Riemann mapping Theorem asserts that any non-empty, simply connected open set in the complex plane which is not the entire plane can be mapped one-to-one conformally onto the open unit disk.

¹¹Originally, the word *Privatdozent* refers to an unpaid lecturer who received fees from the students attending his lectures.

months working on his Habilitation dissertation. This is the memoir which we already mentioned, called *Über die Darstellbarkeit einer Function durch eine trigonometrische Reihe* (On the representability of a function by a trigonometric series) [34]. In this memoir, Riemann obtained the conditions that we now call the Riemann integrability condition, that ensure that a function has an integral.

For his Habilitation, which he completed under Gauss, Riemann was required, as part of his qualifying examination, to deliver a Habilitation lecture. It was customary at Göttingen that the candidate prepares three topics for that lecture. Gauss (being Riemann's mentor) had to choose one of them to hear him lecturing. Riemann prepared two lectures on mathematical physics (mainly electricity and magnetism) and one on geometry. Gauss chose the lecture on geometry, and Riemann delivered his lecture, *Über die Hypothesen welche der Geometrie zu Grunde liegen* (On the Hypotheses Which Lie at the Foundations of Geometry), on June 10, 1854. This lecture is considered to be the founding stone of Riemannian geometry and, beyond that, of all modern geometry, and it revolutionized mathematics. It consisted of two parts. In the first one, Riemann introduced the notion of n -dimensional space, ultimately establishing the foundation for what is known today as a Riemannian manifold.

It is interesting to note that the German word *Mannigfaltigkeit* which Riemann chose to denote a manifold in the mathematical sense, was already used in the German philosophical literature (but not in mathematics), in particular by Kant. Indeed, in the latter's *Kritik der reinen Vernunft* (Critique of Pure Reason), the word *Mannigfaltigkeit*, usually translated into English by “manifold” or “diversity”, designates the raw, unorganised and fragmented data (colors, sounds, textures, and shapes) that our senses receive from the external world before our minds organize it into coherent experiences. Leibniz, who usually wrote in French or Latin (and not in German, which was his mother tongue), in his famous work, *Monadologie* (Monadology), originally written in French, used the word *multiplicité* (multiplicity), to describe a complex plurality co-existing within a single, simple substance, a word which his German translators and commentators transformed again into *Mannigfaltigkeit*. The German philosopher Johann Friedrich Herbart whom Riemann — according to his own account, as we shall recall below — considered himself the heir, in his philosophical writings (late 18th and early 19th centuries), argued that our minds actively construct the concept of space through psychological experience, and used extensively in this context the same word, *Mannigfaltigkeit*, to mean a continuous, structured system of mental concepts. We refer the interested reader to the article *Philosophy behind mathematics* [18] by K. Ohshika on the use of the term *Mannigfaltigkeit* by Riemann, and how the modern precise notion of manifold was later made out of it, through the works of Poincaré, Hilbert, Weyl and others. On the same subject, we also refer to the articles *Mathematical practice as philosophy, with Galois, Riemann, and Grothendieck* by Arkady Plotnitsky [22] and *Le concept de variété et la nouvelle géométrie de l'espace dans la pensée de Bernhard Riemann. L'émergence d'une nouvelle vision des mathématiques et de ses rapports avec les sciences fondamentales* (The concept of manifold and the new geometry of space in Bernhard Riemann's thinking. The emergence of a new vision of mathematics and its relations with fundamental sciences) [3] by Luciano Boi.

It is also interesting to recall that Riemann was familiar, from his religious background and theological studies, with the word *Dreifaltigkeit*, the German word for Trinity, etymologically suggesting a kind of “three-foldedness”. The word *Mannigfaltigkeit* is obtained from the word *Dreifaltigkeit* by replacing the prefix *Drei* (meaning three) by the prefix *Mannig* (which means many).

On February 5, 1854, right after the submission of his Habilitationsschrift, Riemann wrote to his father:

I resumed my investigations into the coherence of the laws of Nature and got so involved in it that I could not tear myself loose. The continuous preoccupation with it has become bad for my health. In fact, right after New Year's, my usual affliction set in with such persistence that I could only obtain relief through the strongest remedies. As a result I felt very ill, felt unable to work, and sought to again put my health in order through long walks.

This letter indicates that Riemann's illness was exacerbated by the heavy workload he placed on himself. In a letter to his brother dated June 26, 1854, he writes, in the same vein:

[...] I had completed my habilitation paper at the beginning of December, submitted it to the dean, and soon after once again turned to my investigation on the coherence of the fundamental laws in physics; also that I so immersed myself in it that when the theme for my examination lecture was posted at the colloquium, I could not immediately tear myself away. Right after, I came down sick, partly, of course, as a result of too much brooding, and partly as a result of sitting a lot in my room during bad weather.

Gauss exerted a profound influence on Riemann. Some historians of mathematics argue that because Gauss was elderly, in poor health and notoriously averse to teaching when Riemann enrolled as his doctoral student, his guidance must have been limited. This assumption, however, contradicts the sheer scope and variety of Riemann's mathematical ideas, many of which he himself explicitly attributed to Gauss's direct mentorship or writings. On this subject, there is, unfortunately, a vast body of literature written by historians of mathematics who are not mathematicians themselves and who fail to understand that, ultimately, a mathematician's influence on his students cannot be measured by the time spent in conversation. Articles that may mislead the reader continue to appear on this topic; see, for example, the recent article [39]. Gauss passed away in 1855, just a year after Riemann completed his Habilitation, but his intellectual imprint on Riemann remained permanent.

Dirichlet moved to Göttingen in 1855, and Riemann was already there. Felix Klein writes in the work we already mentioned [16]:

Riemann was bound to Dirichlet by the strong inner sympathy of a like mode of thought. Dirichlet loved to make things clear to himself in an intuitive substrate; along with this he would give acute, logical analyses of foundational questions and would avoid long computations as much as possible. His manner suited Riemann, who adopted it and worked according to Dirichlet's methods.

Riemann became (what we would call today) an associate professor at Göttingen in 1857 and a full professor in 1859.

During the year 1860, Riemann visited Paris for a few weeks, and he met there, among others, the French mathematicians Joseph Bertrand, Charles Briot, Charles Hermite, Victor Puiseux, and Joseph Serret. Among them, Briot besides being a mathematician, was a physicist and an astronomer. In particular, he devised a formula for light dispersion that bears his name. In mathematics, he worked on Abelian and elliptic functions, a subject which Riemann cherished. Riemann wrote to his sister that he had known and admired Briot for quite some time already, "because of his excellent work".

In June of 1862, Riemann married Elise Koch, a friend of his sisters. The same year, he suffered from an attack of pleurisy. Always in poor health, he spent a large part of the years that followed in Italy, where his doctors had advised him to go because the climate there was favorable to his health. He found financial support and took a leave of absence from Göttingen for these trips to Italy, where he had mathematical colleagues with whom he could discuss. One of these colleagues was Enrico Betti (1823-1892), known to all topologists for the Betti numbers associated with a topological space (the name was given by Poincaré). Betti was also a physicist (we are familiar with the Maxwell–Betti reciprocity theorem) and a specialist in potential theory. He was also among the first to understand the importance of Galois theory. At Betti's suggestion, Riemann was offered a position at the University of Pisa, but he declined it, precisely because of his health. But he spent two consecutive summers in Pisa. André Weil published an article titled *Riemann, Betti and the birth of topology* [44] which contains the transcription of two letters from Betti to his colleague and friend Placido Tardy, dated October 6 and 10, 1863, in which he describes in detail his fascinating conversations with Riemann on the birth of "analysis situs", the word used for the field that became later known as "topology", a field of which Riemann was one of the major founders.

As soon as he returned to Göttingen, Riemann fell ill again, as his body could not tolerate the cold weather. His only child, a daughter called Ida, whom he named after his older sister, was born in Italy.

In early June 1866, Riemann again decided to go to Italy. He wanted to spend a few peaceful months on the shores of Lago Maggiore to recover his forces and finish some works he had started. His departure was delayed a little because of the war; indeed, these were the first days of the war between Prussia and Austria.¹² Riemann reached the Lago Maggiore on June 16, 1866. That was his last trip to Italy. He died there a few days later, on July 20, in Selasca on Lago Maggiore, a hamlet, like the place he was born; he is buried in the nearby cemetery there.

Riemann's life was short, and the same was true for most people in his close family. Along with his mother, three of his sisters and one brother, died before him, deeply shadowing his life. He published his first work at the age of 25 and he died at the age of 39. Thus, his productive period lasted only 15 years. His collected works, which have had a profound and everlasting influence on mathematics, stand in a single slim volume.



Figure 2: Bernhard Riemann in 1863 (at age 37)

During his lifetime, Riemann found it difficult to have friends. The only place where he felt at home was his family's village of Quickborn, where he often used to go, until his father's death in 1855.

1.3 A SHORT SUMMARY OF A LONG STORY

It is impossible to provide a serious account of Riemann's mathematical work within the limited space of a few pages. Instead, in the next section, I will focus on a single topic that is dear to me personally: Riemann surfaces and their moduli. But before that, and since this is a celebration of Riemann's work, I feel committed to make at least a short list of some of his major contributions.

The reader already knows, or has understood by now, that Riemann worked on a variety of topics and that his writings had a wide and profound impact on mathematics. In fact, Riemann's writings had a permanent influence on philosophy and physics as well, and beyond this, on human knowledge. Besides the theory of Riemann surfaces and their moduli, which is the subject of the next section, I mention the following fields on which Riemann had an everlasting influence: differential geometry, topology, the theories of algebraic and complex functions, integration theory, number theory and the zeta function, minimal surfaces, uniformization, the theories of elliptic and Abelian integrals, the hypergeometric series, trigonometric series, electromagnetism, heat propagation, theta functions, Brownian motion, probability theory, quadratic forms, electricity,

¹²The conflict started essentially over a dispute for territory. The key opponents were, on the one side, Austria and its allied German states, including Bavaria, Saxony, Hanover, and Württemberg, and on the other side, Prussia and allies, including the Kingdom of Italy.

gravitation, electrodynamics, magnetism, the investigations on the nature of space, and many others. Riemann's ideas had a great impact on several 20th century philosophers, including the mathematician-philosophers about whom I will mention more in the later sections: Hermann Weyl, André Weil and Alexander Grothendieck. But one could also talk about Riemann's influence on the mathematician-philosopher René Thom, discussed in several chapters of the book [20], and also on a whole generation of French philosophers who were not mathematicians, including Gilles Deleuze, Félix Guattari, Henri Bergson, Albert Lautman, Jules Vuillemin, and there are others. To talk about Riemann's influence on all of them needs numerous articles.

Finally, let me point out that understanding the relationship between the concepts of discreteness and continuity in mathematics and physics, and more generally the relationship between mathematics and physics, which are topics that were dear to Riemann and which are discussed in some detail in the collective book [13]. For the impact of Riemann on the philosophy of mathematics and physics, I would like to mention the articles [[22, 23, 24, 25, 26]] by Arkady Plotnitsky.

1.4 RIEMANN SURFACES AND THE PROBLEM OF MODULI

Riemann's work on what became known as the theory of Riemann surfaces was partly motivated by the desire to find a domain of definition for an algebraic multi-valued complex-analytic function (like $z \mapsto \sqrt{z}$) on which this function becomes single-valued. (As a matter of fact, this is the origin of the word “uniformization” as it was used by Riemann).¹³ The study of Riemann surfaces is also closely related to, and sometimes identified with, the theory of functions of a complex variable, which includes, as a special subfield, that of algebraic functions. This is one of the far-reaching subjects of Riemann's investigations, and its development is one of the most important achievements of nineteenth century mathematics.

The main works of Riemann on this theory is his doctoral dissertation, *Grundlagen für eine allgemeine Theorie der Functionen einer veränderlichen complexen Grösse* (Foundations for a General Theory of Functions of a Complex Variable) [28] (1851) and his memoir *Theorie der Abel'schen Functionen* (The theory of Abelian functions) [29] (1857). On Riemann's dissertation, Ahlfors writes [1]:

This paper marks the birth of geometric function theory [...] Very few mathematical papers have exercised an influence on the later development of mathematics which is comparable to the stimulus received from Riemann's dissertation. It contains the germ to a major part of the modern theory of analytic functions, it initiated the systematic study of topology, it revolutionized algebraic topology, and it paved the way for Riemann's own approach to differential geometry. [...] The central idea of Riemann's thesis is that of combining geometric thought with complex analysis.

In Riemann's doctoral dissertation, we find for the first time the concept of a Riemann surface, the first uniformization theorem (the “Riemann mapping theorem”), and the introduction of potential theory in the theory of Riemann surfaces (the Dirichlet principle). Ahlfors writes in [1, p. 4]: “Riemann virtually puts equality signs between two-dimensional potential theory and complex function theory.” We also find, in this thesis, several fundamental topological concepts, including the notions of connectedness, of simple-connectedness, of degree of connectivity, of coverings and of genus of a surface. We also find there the first version of the Riemann–Roch theorem, the first equivalence relation between Riemann surfaces (in the form of birational equivalence), the first developments of the theory of theta functions, of the Jacobian, of Abelian integrals and of the characterization of these integrals by their period matrices. Furthermore, Riemann stated there for the first time that the “number of moduli” of the set of equivalence classes of a closed surface of genus $g \geq 2$ is $3g - 3$. A central question, which remained open for several decades, was to give a precise meaning to this moduli count, and the effort to answer it gave rise to the

¹³I learned from Yi Huang that the Chinese translation of uniformization is precisely “to render single-valued”.

birth of the theory of moduli and Teichmüller spaces, and to works on this topic by Weierstrass, Schwarz, Dedekind, Klein, Poincaré, Hilbert, Fricke, Koebe, Brouwer, Weyl, Torelli, Siegel, Teichmüller, Weil, Grothendieck, Ahlfors, Bers, Mumford, Deligne, Thurston, McMullen, Witten, Yau, Kontsevich, Mirzakhani and many others.

Regarding Riemann's memoir on Abelian functions, let me quote André Weil, from a letter to his sister Simone Weil, dated March 26, 1940, published in his *Collected Papers* [48, Vol. 1, p. 244-255] and in his sister's *Collected Works* [49, t. VII, vol. 1] He writes the following, talking about algebraic functions of one variable:

It is generally believed that there is nothing left to do on algebraic functions of one variable, because Riemann, who discovered almost everything we know about these functions (I am excepting the works of Poincaré and Klein on uniformization, and those of Hurwitz and Severi on correspondences) did not leave for us any statement of a big problem that concerns them. I am without doubt one of the most knowledgeable persons on this subject; certainly because I had the good fortune (in 1923) to learn it directly from Riemann's writings, whose memoir is of course one of the greatest things that a mathematician has ever written; there is not a single word there that is not of consequence. [...] I am not, of course, foolish enough to compare myself to Riemann, but to add even the slightest thing to Riemann is already, as they would say in Greek, to accomplish something — even if, in doing so, one draws (whether explicitly or implicitly) on the work of Galois, Poincaré, or Artin.

It would be unreasonable to try to develop all these topics (even one of them) in a short article. Let me only say a few more words on the concept of Riemann surface.

Riemann viewed such a surface as a branched cover over the sphere, but above all, as a new space on which one can do function theory. From the beginning of his dissertation, we read:

We restrict the variables x, y to a finite domain by considering as the locus of the point O no longer the plane itself but a surface T spread over the plane. We admit the possibility ...that the locus of the point O is covering the same part of the plane several times.

This notion, as an abstract surface, remained misunderstood for decades, even among prominent mathematicians, who continued to think of surfaces embedded in the three-dimensional Euclidean space with self-intersection, which is not the way Riemann conceived them; see, for example, Figure 4, taken from the famous treatise by Goursat and Appell, *Théorie des fonctions algébriques et de leurs intégrales — Étude des fonctions analytiques sur une surface de Riemann* (Theory of algebraic functions and their integrals: A study of analytic functions on a Riemann surface) [2] (1895).

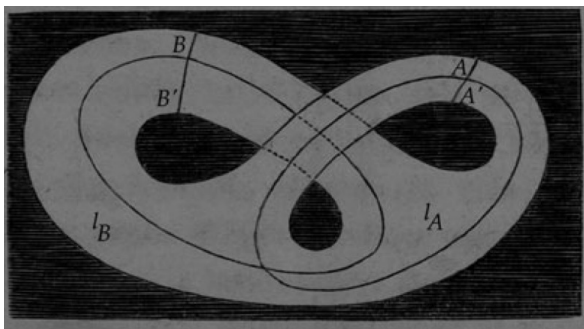


Figure 3: A Riemann surface, drawing from Riemann's paper on Abelian functions (1851)

Regarding the theory of functions of a complex variable, Klein, in his article *Riemann und seine Bedeutung für die Entwicklung der modernen Mathematik* (Riemann and his significance for the development of modern mathematics) (1895) [15], attributes the beginning of this theory to Cauchy. He writes:

The founder of this theory is the great French mathematician Cauchy; but only later, in Germany, did this theory assume its modern aspect which has made it the central point of our present views of mathematics. This was the result of the simultaneous efforts of two mathematicians whom we shall have to name together repeatedly, — of Riemann and Weierstrass.

It is interesting to see that Klein writes after this, in a footnote: “In the text I refrained from mentioning Gauss, who being in advance of his time in this and in other fields, anticipated many

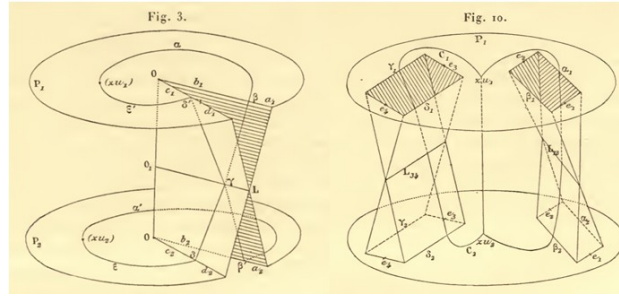


Figure 4: A Riemann surface, from the treatise by Appell and Goursat [2] (1895), imagined as a surface embedded with self-intersections in Euclidean space



Figure 5: Felix Klein (1849-1925) considered that his main mission as a mathematician was to explain and promote Riemann's work

discoveries without publishing what he had found. It is very remarkable that in the papers of Gauss we find occasional glimpses of methods in the theory of functions which are completely in line with the later methods of Riemann, as if unconsciously a transfer of leading ideas has taken place from the older to the younger mathematician”.

In his work on Riemann surfaces, Riemann made the relation between Riemann surfaces and potential theory. Let me quote Klein about this, from the work we just mentioned [15]:

It should also be observed that potential theory, which in our day, owing to its importance in the theory of electricity and in other branches of physics, is quite universally known and used as an indispensable instrument of research, was at that time in its infancy. It is true that Green had written his fundamental memoir as early as 1828; but this paper remained for a long time almost unnoticed. In 1839, Gauss followed with his researches. As far as Germany is concerned, it is mainly due to the lectures of Dirichlet that the theory was farther developed and became known more generally; and this is where Riemann finds his base of operation.

Klein talks here about the importance of potential theory in electricity and in other branches of physics. Riemann never separated physics from mathematics, and in fact, he never separated mathematics from physics and philosophy. This is what I want to communicate in the next section. To end the present section, let me quote Klein again, from an address he delivered in Vienna on September 27, 1894: “Riemann was one of these out-of-the way scholars who let their profound thoughts ripen, silently, in their mind.”

1.5 MATHEMATICS, PHYSICS, AND PHILOSOPHY

Riemann's work had a profound and permanent impact, not only on mathematics, but also on mechanics, electromagnetism, the theory of heat, electricity, acoustics, and other fields of physics.

There is also a collection of articles by Riemann on philosophy, psychology, metaphysics, and gnoseology (the philosophical study of the origin, nature, scope, and limits of human knowledge), edited by the mathematician Heinrich Weber (1842-1913) in Riemann's *Collected Works* [33]. In this section, I will discuss briefly some ideas from physics and philosophy, and sometimes make relations with mathematics.

For Riemann, mathematics and physics cannot be separated. His mathematical papers contain ideas that stem from physics. Many of the problems he addressed were motivated by physics, and on several occasions he employed lines of reasoning derived from physics. His work had a definite impact on this field. Hermann Weyl and Albert Einstein, to mention only two names that are familiar to every mathematician, based their physical theories on Riemann's mathematical work. Klein, in his address delivered at the 1894 general session of the *Versammlung Deutscher Naturforscher und Ärzte* (Meeting of the German naturalists and physicians) which was held in Vienna, on September 27, 1894, cf. [14], writes:

I must mention, first of all, that Riemann devoted much time and thought to physical considerations. Grown up under the tradition which is represented by the combinations of the names of Gauss and Wilhelm Weber, influenced on the other hand by Herbart's philosophy, he endeavored again and again to find a general mathematical formulation for the laws underlying all natural phenomena. [...] The point to which I wish to call your attention is that these physical views are the mainspring of Riemann's purely mathematical investigations.

Besides physics, philosophy is always in the background of Riemann's thought. For him, mathematics, physics, and philosophy constitute three facets of the same body of knowledge. His Habilitation lecture *Über die Hypothesen, welche der Geometrie zu Grunde liegen* (On the hypotheses that lie at the bases of geometry) [35] is a merging of philosophy, geometry and physics. The fundamental questions that he explicitly addresses there are concerned with space, form, dimension, magnitude, the infinite and the infinitesimal, the discrete and the continuous. These are precisely the questions that obsessed the ancient Greek philosophers, starting with the Milesians and the Pythagoreans, and passing through Plato, Aristotle, Archimedes and the other mathematicians-philosophers from Antiquity. Klein, in his *Development of mathematics in the 19th century* [12, p. 235], reporting on some of Riemann's ideas from what became known as his *Nachlass*,¹⁴ writes:

Riemann thinks of space as being filled with continuous matter [Stoff], which transmits the effect of gravity, light, and electricity. He has throughout the idea of a temporal extension of process. A remark on this topic is found in a personal letter from Gauss to Weber — with an express request for complete secrecy. And now I again ask, how did these things come to Riemann? It is just mystical influence, which cannot be defined and yet cannot be clearly grasped, of the general atmosphere of a receptive spirit.

From a purely mathematical perspective, the question of whether space is continuous or discrete has had enormous consequences: it led, for instance, to the foundation of arithmetic geometry. We will return to this topic at the end of this section, where we discuss the work of Grothendieck, as well as in the final section of this article, in connection with the work of André Weil and, once again, that of Grothendieck. This “analogy” between, on the one hand, the continuous and the discrete, and on the other hand, analysis and arithmetic, is one of the fundamental ideas expressed by André Weil in his famous letter to his sister, dated March 26, written from prison [45, Item 1940a], which we mention without being able to say more about it due to lack of space.

It follows from Riemann's correspondence that as a student, in Göttingen and Berlin, he attended more courses and seminars on (theoretical and experimental) physics than on mathematics, and this interest in physics remained constant during his lifetime. Since his early twenties, he tried to develop a theory that would unify electricity, magnetism, light and gravitation — the same quest that Poincaré, Lorentz and Einstein addressed after him, culminating in the theory of

¹⁴This is the collection of manuscripts, notes and correspondence that he left after his death in 1866.

general relativity. The content of one of his papers, *Ein Beitrag zur Electrodynamik* (A contribution to electrodynamics) [36], published posthumously, is part of his search for the unification of the various forces of nature. One may also mention in this respect his essay titled *Gravitation und Licht* (Gravitation and light) [33, p. 532-538], whose subject is the connection between gravitation and light. In his lectures *Schwere, Electricität und Magnetismus* (Gravity, electricity and magnetism), edited by his student Karl Hattendorf in 1880 [37], he establishes necessary and sufficient conditions under which Hamilton's principle is satisfied. This is the problem of motion of a free system subject to forces that depend on position and motion but that are time-independent. Émile Picard, in Chapter VII of his famous *Traité d'analyse* (Treatise on analysis) [21], has a chapter called *Attraction and potential*. He uses there (p. 167) a transformation from Riemann's posthumous memoir *Gravity, electricity and magnetism* which we just mentioned. Let us also note that the physicist and mathematician James Clerk Maxwell discusses Riemann's theory of electrodynamics in his *Notiz über die elektromagnetische Theorie des Lichts* (Note on the electromagnetic theory of light), an appendix to his paper [17]. In the paper *Riemann on Geometry, Physics, and Philosophy: Some Remarks* [11] Jeremy Gray, discusses the general question of the interaction between the mathematical ideas expressed in Riemann's Habilitation lecture *On the hypotheses that lie at the bases of geometry* and physics and philosophy.

In a note Riemann wrote about his own work [12, p. 233], we read:

My main work concerns a new conception of the known works of nature, their expression by means of other basic concepts, whereby it became possible to use the experimental data on the reciprocal actions between heat, light, magnetism and electricity to investigate their connections with each other. I was led to this mainly through studying the works of Newton, Euler, and — from another aspect — Herbart.

The name of Johann Friedrich Herbart (1776-1841), which appears at the end of this quote, is poorly known to mathematicians, but is quoted several times by Riemann in his writings. A few words on Herbart are in order.

Herbart studied philosophy in Jena under Johann Gottlieb Fichte (1762-1814), one of the founders of the philosophical movement known as German idealism, which had its origins in the theoretical and ethical writings of Kant. Herbart studied in Göttingen where he received his doctorate and habilitation, and he later taught there pedagogy and philosophy, before he was offered the chair formerly held by Kant in Königsberg. His philosophy relies on the theory of monads of Leibniz, another mathematician-philosopher. Herbart wrote in 1824 a treatise titled *Psychologie als Wissenschaft, neu gegründet auf Erfahrung, Metaphysik und Mathematik* (Psychology as a science newly founded on experience, metaphysics and mathematics). He made use of infinitesimal calculus in his research on psychology, which he considered as a science based at the same time on experimentation, mathematics and metaphysics. He drew a parallel between this new field and physics, in the way Newton conceived it. Sigmund Freud was profoundly influenced by Herbart. There are several sets of notes written by Riemann and preserved in the Riemann archives in Göttingen, which concern the philosophy of Herbart. These notes show that Riemann was highly influenced by the philosopher, in respect of epistemology and the philosophy of science, and in particular for his ideas on space. Most of all, Herbart and Riemann addressed the same questions. For instance, both of them addressed the question of whether or not our knowledge of space, time, and motion is generated by experience. In a serious discussion of Riemann's philosophy of space, one should mention Kant. But let me quote instead a passage from another mathematician-philosopher, Alexander Grothendieck, from his vast autobiographical, poetic, and philosophical narrative *Récoltes et Semailles: Réflexions et témoignage sur un passé de mathématicien* (Harvests and Sowings: Reflections and testimony about a past as a mathematician) [12]. The passage is worth quoting in its entirety because it contains a certain number of great ideas. Grothendieck writes [12]:¹⁵

¹⁵The English translation is mine.

It must have been fifteen or twenty years ago now, as I was leafing through the modest volume containing Riemann's complete works, that I was struck by a remark he made "in passing." In this remark, he observes that the ultimate structure of space might well be "discrete", and that the "continuous" representations we form of it may constitute a simplification (perhaps excessive, in the long run, ...) of a more complex reality; that for the human mind, the "continuous" was easier to grasp than the "discontinuous," and that it therefore serves as an "approximation" for us to comprehend the discontinuous. This is a remark of surprising insight coming from a mathematician, at a time when the Euclidean model of physical space had never yet been questioned; in a strictly logical sense, it is rather the discontinuous that has traditionally served as a technical approach to the continuous.

The developments in mathematics over the past few decades have, in fact, revealed a much closer symbiosis between continuous and discontinuous structures than was imagined in the first half of this century. Be that as it may, finding a "satisfactory" model (or, if necessary, a set of such models that "fit together" as satisfactorily as possible ...), whether "continuous", "discrete," or of a "mixed" nature — such work will surely require great conceptual imagination and a consummate flair for grasping and uncovering mathematical structures of a new kind. This kind of imagination or "flair" strikes me as rare, not only among physicists (where Einstein and Schrödinger seem to have been among the few exceptions), but even among mathematicians (and here I speak from firsthand experience). To sum up, I foresee that the expected renewal (if it is yet to come ...) will come more likely from a mathematician at heart, well-informed about the major problems of physics, than from a physicist. But above all, it will require a person with "philosophical openness" to grasp the crux of the problem. This problem is by no means technical in nature, but rather a fundamental problem of the "philosophy of nature."

Arkady Plotnitsky, in his recent paper *Continuity and discontinuity in, and between, mathematics and physics* [23], considers the question of discreteness and continuity in modern mathematics and in physics, including this question in the later problems addressed by quantum field theories and quantum mechanics. Among other things, he discusses there the idea advanced by Riemann, followed by Grothendieck, that the continuous may serve as an approximation of the discontinuous.

In the next section, and in guise of a conclusion to this paper, I would like to talk briefly about Riemann's influence on some of the greatest mathematicians of the last century, and I chose Weyl, Weil, and Grothendieck, all three being, each one in his own way, a Riemann heir.

1.6 WEIL, WEYL, GROTHENDIECK AND RIEMANN

I started this tribute to Riemann by mentioning his predecessors, and showing how he quoted them in his works. I can think of no better way to conclude the same tribute to Riemann than to quote some major twentieth-century mathematicians who were familiar with his work, who understood it deeply, who incorporated it into their own work, and showing the way they quoted him. I am going to take this opportunity to say a few words about each of these mathematicians. This will be also an occasion to talk about some of Riemann's results that were not mentioned in the previous sections.

I start with Hermann Weyl (1885-1955), who is one of the most prominent theoretical physicists of the twentieth century. He was the first to realise a synthesis between general relativity and electromagnetism. He introduced the notion of gauge invariance and several other fundamental ideas in physics.

Weyl was always engaged with Riemann's mathematical, physical and philosophical legacy, frequently referring to him as a bridge between geometry and physics. His work *Raum · Zeit · Materie: Vorlesungen über allgemeine Relativitätstheorie* (Space - Time - Matter: Lectures on General Relativity) [50] shows that Riemannian geometry is the most natural, and often the necessary, framework for several branches of modern physics. To emphasize the naturalness of this field, Weyl makes a comparison between the fact of moving from the flat Euclidean geometry to the curved Riemannian and the fact of moving from the action-at-a-distance physics to field theory. He writes in the work we just mentioned that

the approach of understanding the world from its behavior in the infinitesimally small is the driving epistemological motive of near action physics and Riemannian geometry, but it is also the driving motive in the rest of Riemann's magnificent life's work, which is primarily directed towards complex function theory.

In the same work, Weyl mentions the mathematicians and physicists who built on Riemann's ideas. He writes:

After Riemann had made known his discoveries, mathematicians busied themselves with working out his system of geometrical ideas formally; chief among these were Christoffel, Ricci, and Levi-Civita. Riemann [...] clearly left the real development of his ideas in the hands of some subsequent scientist whose genius as a physicist could rise to equal flights with his own as a mathematician. After a lapse of seventy years this mission has been fulfilled by Einstein.

In Section 1.4 of the present essay, we have discussed Riemann's Complex function theory and his introduction of the concept of Riemann surface. It was Hermann Weyl, in his work *Die Idee der Riemannschen Fläche* (the concept of Riemann surface) [51], who, relying on Riemann's intuitive ideas, formulated a precise definition of a surface equipped with an atlas, that is, built from local coordinate systems, paving the way for the modern definition of a manifold.



Figure 6: Hermann Weyl (1885-1955) built rigorous foundations for Riemann's visionary yet informal ideas on function theory, topology and geometry in his landmark 1913 book, *The Concept of a Riemann Surface*

From Hermann Weyl, we pass to André Weil (1906-1998), the founder of modern algebraic geometry who is also closely associated in several respects with Riemann.

Weil started reading Riemann's works while he was a student at the École Normale Supérieure, which he entered at the very precocious age 16. He writes in his autobiographical essay, *Apprenticeship of a mathematician* [47]:

That same year,¹⁶ I began to read Riemann. [...] During my year of instruction in philosophy,¹⁷ I had also been struck by a phrase of Poincaré's which expresses no less an extreme position: "The value of civilizations lies only in their sciences and arts." With such ideas in my mind I had no choice but to dive headlong into the works of the great mathematicians of the past, as soon as they were materially and intellectually within my grasp. Riemann was the first; I

¹⁶That is, the year he entered École Normale.

¹⁷This was André Weil's final year of high school, which he completed at age fourteen.

read his inaugural dissertation and his major work on Abelian functions. Starting out thus was a stroke of luck of which I have always been grateful. These are not hard to read as long as one realizes that every word is loaded with meaning: there is perhaps no other mathematician whose writing matches Riemann's for density.

There is no doubt about the fact that if there is one open question that André Weil would have liked to settle, then this would be the Riemann hypothesis. This problem, formulated in 1859 by Bernhard Riemann in his eight-page article *Über die Anzahl der Primzahlen unter einer gegebenen Grösse* (On the Number of Primes Less Than a Given Magnitude) [32] states that the infinite series $\zeta(s) = \sum_{n \geq 1} n^{-s}$ (known after him as the Riemann zeta function) has all its nontrivial zeros on the line $\Re(s) = 1/2$.

Weil spent a substantial part of his mathematical life in trying to prove the Riemann hypothesis. Because of the lack of space, it is not possible to enter here into more details about this statement, except for saying that the study of the properties of this function goes back to Euler, who was also the first to discover the relation between this function and the distribution of prime numbers. The reader interested in a discussion of and in references to Euler's work on this function may consult [19, §6].

André Weil formulated and proved an analogue of the Riemann hypothesis for algebraic curves over finite fields. Again, it is not possible to enter here in the details, but we can say it in a minimal amount of words. The statement is equivalent to showing that the number of rational points on a curve defined over a number field is closely related to the size of the field. Weil announced this result in two notes published in 1940 and 1941, and he wrote the detailed proofs in papers and books he published in 1942, 1946 and 1948, see [40, 41, 42, 43]. It is generally admitted that in so doing, Weil established the foundations of modern Algebraic Geometry.

In an interview of André Weil [46, p. 10], conducted in October 1991 (Weil was 86 years old), in response to the question “Mathematically speaking, do you have any hopes or regrets?”, Weil replied: “When I was young, I hoped to prove the Riemann Hypothesis. When I got a little older, I still hoped to be able to read and understand a proof of the Riemann Hypothesis. Now, I would be quite content just to learn that a proof exists.”

A recent article by Alain Connes starts with the following [6]: “The Riemann Hypothesis stands as perhaps the most famous unsolved problem in mathematics, having resisted all attempts at proof since Bernhard Riemann casually remarked in his 1859 memoir that ‘sehr wahrscheinlich’ (very probably) all nontrivial zeros of his zeta function lie on the critical line $\Re(s) = 1/2$.”

André Weil's work on the Riemann Hypothesis for curves over finite fields is an illustration of Bernhard Riemann's original idea from a shift between the continuous geometric and analytical setting to the discrete algebraic setting. Another illustration of the same idea is the famous Riemann–Roch–Grothendieck theorem, which we briefly introduce below. But first, we say a few words on the classical Riemann–Roch theorem



Figure 7: André Weil
(1906–1998)

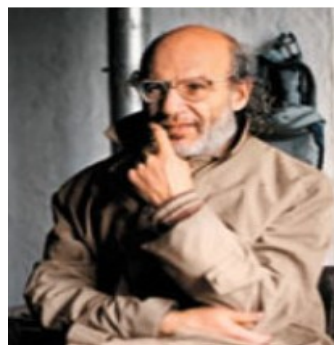


Figure 8: Alexander Grothendieck
(1928–2014)

The origin of the Riemann–Roch Theorem can be traced back to Riemann's existence theorem, which asserts the existence of meromorphic functions on Riemann surfaces. Providing deeper

insight, the classical version of the Riemann–Roch theorem focuses on the dimension of the space containing these functions on a compact Riemann surface, specifically, with poles of a certain order at some prescribed set of points. This theorem is stated as a formula that gives this dimension in terms of the surface’s genus and the total order at its ramified points. Thus, the theorem provides a connection between topological structures and analytical concepts. There are several versions and generalizations of this theorem, many developments and many applications, including the Hirzebruch–Riemann–Roch theorem which applies to higher-dimensional complex spaces, and the Grothendieck–Riemann–Roch theorem on which we shall say a few words. Atiyah–Singer’s index theorem, discovered in 1963, is also considered as a generalization of the Riemann–Roch theorem.

Speaking of Grothendieck, I cannot help but recall his involvement in a radical ecological movement, particularly through the environmental and political group *Survivre et vivre*, which he co-founded, his anti-war activism (this was at the time of the U.S. intervention in Vietnam), and his resignation from his professorship at IHES after discovering that the Institute was receiving funding from the Ministry of Defense.

Grothendieck obtained a very general version of the Riemann–Roch theorem, formulated in the language of categories and functors, which generalizes the Riemann–Roch theorem for algebraic varieties of arbitrary dimension and defined over arbitrary ground fields. This was one of Grothendieck’s major discoveries. In his autobiographical work *Récoltes et Semailles* which we already mentioned [12], he writes: “The year 1957 is the one when I was led to extract the theme ‘Riemann–Roch’ (Grothendieck’s version) which overnight consecrated me as ‘great star.’” Grothendieck’s version of the Riemann–Roch theorem was the starting point of K-theory. In the section called *La vision — ou douze thèmes pour une harmonie* (The vision — or twelve themes for a harmony) of *Récoltes et semailles*, Grothendieck considers the theme he calls the Riemann–Roch–Grothendieck Yoga as one of the twelve themes which he describes as his “great ideas” (*grandes idées*). There is also a discrete Grothendieck–Riemann–Roch version, proved by Grothendieck and whose full version for general schemes remains an open research field rather than a theorem.

I would like to conclude this article by quoting three passages from Grothendieck’s *Récoltes et Semailles*,¹⁸ the work in which he presents his vision of what a mathematician is and what type of mathematician he himself belongs to. In doing so, he mentions at several places the name of Riemann, not only for his mathematical results, but also as the prototype of a mathematician, and for his way of understanding mathematics.

The first passage is extracted from §6.4 of this work, a section titled *Dream and proof*. Grothendieck describes there Riemann as the model of a young student who can understand what topology is. He writes:

When I talk to students about the topology of a sphere, and the shapes that result from a sphere by adding handles, things that do not surprise young children but confuse them because they think they know what “math” is, the first spontaneous reaction I get is: “But that’s not math! Math, of course, is the Pythagorean theorem, the altitudes of a triangle, and quadratic polynomials ...” These students aren’t any dumber than you or me; they are reacting the same way that, from time immemorial right up to the present day, even all the mathematicians in the world have reacted, except for people like Pythagoras or Riemann and perhaps five or six others.

The second excerpt is taken from Section 2.5 of the same book, a section titled *The heirs and the builder*:

As for me, I feel I belong to the lineage of mathematicians whose spontaneous calling and joy is to constantly build new houses. Along the way, they cannot help but also invent and gradually

¹⁸The quotations from Grothendieck in this paragraph are taken from a text titled “A Stroll Through a Work, or The Child and the Mother,” which serves as an introductory essay to the entire *Récoltes et Semailles*, in which he reflects on his life and on his spiritual experience, sharing images of the landscapes of his internal world, of his vision of mathematics and of his ideas on the nature of discovery, and examining with a critical eye the scientific community of which he was part.

shape all the tools, utensils, furniture, and instruments required — both to build the house from the foundation to the roofline, and to abundantly furnish the future kitchens and workshops, and to set up the house so that it is livable and comfortable. Yet, once everything is in place, down to the very last gutter and the very last stool, it is rare for the craftsman to linger long in these places, where every stone and every rafter bears the mark of the hand that worked and set it. His place is not in the tranquility of ready-made worlds, however welcoming and harmonious they may be, whether they were arranged by his own hands or by those of his predecessors. Other tasks are already calling him to new construction sites, driven by the imperious urge of needs that perhaps only he can clearly sense, or (even more often) by anticipating needs that only he can foresee. His place is out in the open air. He is a friend of the wind and does not fear working alone, for months and years — and, if necessary, for a lifetime — unless a welcome replacement comes to his aid. He has only two hands, like everyone else, of course, but two hands that at every moment instinctively know what they must do, that shrink from neither the heaviest tasks nor the most delicate ones, and that never tire of discovering and rediscovering those countless things that constantly call upon them to be known. Two hands are meagre, perhaps, for the world is infinite. They will never exhaust it! And yet, two hands are a great deal ...I, who am not well-versed in history, if I were to name mathematicians in that tradition, the names that immediately come to mind are those of Galois and Riemann (in the last century) and that of Hilbert (at the beginning of the present century).

Finally, in Section 4.2.3 of the same book, in a paragraph titled *The end of a secret*, Grothendieck uses a poetic metaphor to describe the brilliance, purity, and immense creative potential he had discerned in one of his students, who immediately reminded him of Riemann.¹⁹ He writes²⁰:

I had the privilege of witnessing the first blossoming of a child's enthusiasm, carrying the promise of a vast unfolding. The blossoming I had in mind is the fruit of innocence, eager for knowledge and always ready to delight in the beauty of things both small and great in this inexhaustible world, or in a particular part of that world (such as the vast world of mathematics ...). He alone possesses the power of profound renewal, whether it be the renewal of the self or that of knowledge of the things of this world. It is this power that, it seems to me, found its fullest realization in the modest figure of Riemann. This true fulfillment is foreign to contempt: contempt for others (those we feel are far beneath us ...), or for things too "small" or too obvious for us to deign to take an interest in, or for those we deem to fall short of our legitimate expectations; or even contempt for a certain *dream*, perhaps, which speaks to us insistently of the things one professes to love. It is foreign to contempt, just as it is foreign to the concept that fuels it.

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¹⁹Grothendieck is talking here about his student Pierre Deligne

²⁰Since there are several versions of *Récoltes et Semailles* on the internet, which are sometimes different in length, let me give the address of the link. I am using here: <https://webusers.imj-prg.fr/~leila.schneps/grothendieckcircle/RetSbis.pdf>

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2. A Peep into History of Mathematics

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In this entry we review two papers on historical themes, one relating to algebra in ancient and medieval China and another on Nārāyaṇa Paṇḍita's constructions of magic squares.

Wanwan Huang On the development of equations in ancient China, *Journal of Humanistic Mathematics* 15 (2025), no. 1, 165 - 191.

Linear algebra, or the theory of linear equations, is known to have made its early appearance in China a few centuries before the onset of the common era, in the work *Suanshu shu*, and later in an evolved form in *Jiuzhang suanshu* (Nine Chapters on the Mathematical Art), written in the early Eastern Han Dynasty (25-220 CE). The author notes that there were three more stages of development of algebra, or equations, in pre-modern China. The second and third were occasioned by the works *Sunzi suanjing* (ca.4th century) and *Zhang Qiujiang suanjing* (466-485 CE), which were later followed by development of a matured theory of equations during the Song Dynasty (960-1279 CE).

The article under review describes the major features of the four phases mentioned, together with some crucial examples of problems they dealt with. It is noted that the theory that emerged during the early phase, did not quite flourish in the subsequent centuries. The author suggests that the rise in usage of abacus, which served as an efficient computational tool but could not deal with unknown, as one of the causes for this. Interestingly the second phase features what we now call the Chinese remainder theorem, though in a limited way; it was developed into a complete procedure by Qin Jiushao (13th c.). Works in the third phase specialize in practical applications such as measurement, textiles, exchange, taxation, smelting, civil engineering, and interest. The fourth phase witnessed a vast improvement in being able to solve equations, including quadratic, as well as some cubic and biquadratic equations, involving also some aspects of Geometric algebra. The article especially discusses, towards the end, the work of Tian Yuan in this respect.

Mandadi Aruneshwar Reddy, Sooryanarayan D.G., and Ramasubramanian K. *Turagagati* method for 4×4 pandiagonal magic squares by Nārāyaṇa Paṇḍita, *Ind. J. of Hist. Sci.* 59 (2024), 123-142.

A magic square of order n is an arrangement of n^2 distinct numbers in a $n \times n$ grid such that the sum of the numbers along any row or column is the same; it is said to be a *pandiagonal* magic square if the sum along either of the diagonals, as well as along any diagonal across the torus of the square (proceeding from any entry along one of the diagonals and continuing on the opposite side upon meeting the edge) also add up to the same number. Construction of magic squares has a long history in India, going back at least to Nāgārjuna from the 1st century, for $n = 4$, with many more exponents involved during the later period. Chinese, Hebrew, Persian and Arabic scholars have also contributed to the literature on magic squares at different times.

A detailed mathematical treatment of the topic was given by Nārāyaṇa Paṇḍita in his work *Gaṇitakaumudī*, composed in 1356-57 (śaka year 1278 is mentioned by the author), devoting to it a chapter, with 77 verses. In the work he describes in particular a procedure for constructing 4×4 pandiagonal magic squares, through a procedure for filling the entries of the grid, moving on successively through steps like a horse's move (*Turagagati*) in the game of chess (two squares in one direction and combined with one square shift in the other direction). The authors note that Nārāyaṇa Paṇḍita methodically introduces all topics that he discusses.

The article begins with a discussion on the history of the topic and various background aspects of the work *Gaṇitakaumudī* and its author, and then proceeds to give a detailed exposition on the *Turagagati* procedure, recalling the relevant verses, followed by translation and interpretation of the verses, and step by step description of the indicated procedure with the aid of illustrative figures. There is also a discussion on various mathematical aspects related to the procedure. It would also be of interest to note here that recently, in a follow-up paper (same journal, Vol 61, pp. 1-8), building on the explorations in the present paper the authors describe a simple procedure which produces 4×4 pandiagonal magic squares in just 4 steps. □ □ □

3. What is Happening in the Mathematical World?

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3.1 THE SUMMER OF SYNTHESIS: MACHINE INTELLIGENCE AND DEEP MATHEMATICAL PROOFS



The quarter spanning from June 2026 to September 2026 will stand in mathematical records as an extraordinary inflection point. While the opening months of 2026 demonstrated that machine reasoning could independently discover novel counterexamples to Paul Erdős's combinatorial geometry conjectures and settle isolated questions, this past summer has witnessed something vastly more profound: the deep integration of machine reasoning into foundational problems alongside an

extraordinary explosion of pure human mathematical breakthroughs.

Across research institutions in Paris, Princeton, Cambridge, Zurich, and Beijing, mathematicians have stopped viewing artificial intelligence merely as a curiosity. Instead, high-capacity neural operators and automated formal proof engines like *Lean 4* have joined forces with human analysts to tackle two of the most formidable intellectual mountains: the fluid blow-up singularities of the Navier-Stokes equations and the machine-certified verification of Andrew Wiles' proof of Fermat's Last Theorem.

3.1.1 A Solution of 3-Dimensional Navier-Stokes Equation constructed by OpenAI that Blowups in Finite Time

September 8, 2026 will go down as a milestone in artificial intelligence. For the first time, an AI model was used to solve one of mathematics' most famous problems - a Millennium Problem. OpenAI announced that a group of 10,000 agents had produced a solution to a problem known as the **Navier-Stokes Existence and Smoothness problem** which is one of the seven Millennium Prize Problems designated by the Clay Mathematics Institute in May 2000. In his official problem description for the Clay Mathematics Institute, Charles Fefferman formalized this problem by framing it around specific mathematical alternatives for an incompressible fluid in three-dimensional space, as follows:

Consider a viscous, incompressible fluid flowing in three-dimensional space ($\mathbb{R}^3$) or on the three-dimensional torus ($\mathbb{T}^3$), subject to a smooth external force $f(x, t)$. The motion of such a fluid is governed by the Navier-Stokes equations in terms of the velocity vector field $u(x, t)$, scalar pressure $p(x, t)$ and the kinematic viscosity (a positive constant) v :

$$\partial u / \partial t + (u \cdot \nabla) u = -\nabla p + v \Delta u + f(x, t); \nabla \cdot u = 0 \text{ with initial condition } u(x, 0) = u_0(x),$$

where u_0 is smooth (C^∞), divergence-free vector field. Fefferman notes that the central goal of the Millennium Problem is to prove or disprove one of the following four alternatives:

- (A) There exists a globally smooth, bounded-energy solution u, p on $\mathbb{R}^3 \times [0, \infty)$.
- (B) With $f = 0$, there exists a globally smooth, bounded-energy solution u, p on $\mathbb{R}^3 \times [0, \infty)$.
- (C) There exist smooth $u_0(x)$ and $f(x, t)$ such that no global smooth solution with bounded kinetic energy exists.
- (D) There exist smooth $u_0(x)$ and $f(x, t)$ as described in (C) in case when $f = 0$.

For nearly two centuries, scientists and mathematicians have used the Navier-Stokes equations to describe the movement of all fluid. The equations, formalized in the 19th century by physicists *Claude-Louis Navier* and *George Gabriel Stokes*, make precise predictions about everything from

how wind flows over an airplane wing or blood pumps through a vein, to how massive swirls and eddies drive the motion of Earth’s oceans and atmosphere.

Despite their universal use in physics and engineering, mathematicians still do not know whether smooth, physically realistic initial conditions can develop a disastrous ‘singularity’: an instant, where fluid velocity, pressure, or vorticity spike to infinity in a finite amount of time.

In late July 2026, an interdisciplinary consortium of mathematicians and computational scientists from Caltech, NYU’s Courant Institute, and Princeton unveiled an autonomous system named ‘Neuro-Singularity-Explorer’ (NSE-26). Traditional supercomputers have struggled for decades because numerical grids break down when structures shrink toward zero thickness. NSE-26 bypassed this by combining Fourier Neural Operators (FNOs) with Physics-Informed Neural Networks (PINNs) to explore the infinite-dimensional function space of fluid geometries, searching specifically for ‘self-similar collapsing profiles’ - vortex shapes that preserve their geometry while rapidly imploding.

The neural operator discovered an astonishing, counterintuitive configuration that human researchers had never conceived. For every positive viscosity, they construct a solution of the three-dimensional incompressible Navier-Stokes equations that starts from rest and develops unbounded velocity in finite time while maintaining uniformly bounded kinetic energy. This establishes a special case of alternative (C) in the Millennium problem statement as mentioned above.

Crucially, the advance did not remain a mere numerical simulation. Throughout August 2026, human mathematicians converted the AI’s numerical profile into a rigorous mathematical ansatz. The work has culminated in a 166 pages long paper entitled “FINITE TIME BLOWUP FOR NAVIER- STOKES” which is published directly via OpenAI’s research portal and hosted on their PDF content delivery network (OpenAI Research PDF). This marks the closest mathematics that has ever come to confirming a true finite-time singularity in 3D Navier-Stokes boundary equations. However, the prize is not yet won. The breakdown occurs near a physical wall, leaving open the question of whether singularities can form in open, borderless fluids.

Source: <https://cdn.openai.com/pdf/32d9f210-8b73-45e0-91bc-82a30aef8a9a/navier-stokes.pdf>

3.1.2 Fermat’s Last Theorem Formalized by AI Agents in Just 11 Days



In 1637, *Pierre de Fermat* wrote his immortal note in the margin of Diophantus’s *Arithmetica*: No three positive integers a, b , and c satisfy $a^n + b^n = c^n$ for any integer $n \geq 3$. His teasing claim that he possessed a ‘truly marvelous proof’ that the margin was too narrow to contain, ignited a 358-year hunt. That hunt terminated in 1994 when *Andrew Wiles* proved the Modularity Theorem for semistable elliptic curves, delivering one of the greatest achievements of human thought.

Yet Wiles’ proof spans over 130 pages of dense, ultra-sophisticated 20th-century algebraic geometry - drawing upon deep results by *Grothendieck*, *Serre*, *Mazur*, *Ribet*, and *Langlands*. Because only a handful of living mathematicians understand every single deduction, and modern research literature occasionally put aside unverified lemmas, *Prof. Kevin Buzzard* of Imperial College, London launched the international ‘**Fermat’s Last Theorem Lean 4 Project**’ in 2024 to convert the entire proof into computer-verified code that a proof assistant kernel can certify with 100% logical perfection.

Human estimates initially suggested the monumental project would take until 2030 or beyond. However, in mid-August this year, the project achieved a spectacular breakthrough via neuro-symbolic auto-formalization tools developed in collaboration with Google DeepMind and Meta AI.

Anthropic, an American artificial intelligence startup and Public Benefit Corporation founded in 2021, provide in [1] some background on Claude’s formalization of Fermat’s Last Theorem in Lean, including a timeline of the proof and verbatim excerpts from its reasoning.

Early in August, a small team at Anthropic kicked off a Claude-driven attempt to formalize Fermat's Last Theorem in Lean. This run used a new harness based on Prove2Me, a platform developed by Anthropic researcher Tianyi Peng's group at Columbia University. Claude ran largely autonomously for eleven days before completing the proof. A team of Claude agents working in parallel wrote the statements, checked one another's statements, and proved them. The proof built on Mathlib (The Lean mathematical library), on the above referred Imperial College London FLT project [2], and on flt-regular [3], a complete formalization of Fermat's Last Theorem for regular primes in Lean. The proof involved 29511 theorems and 13 million lines of Lean code, relied only on Lean's three standard axioms, and left no unproved placeholders (sorry). The full proof can be found at [2]

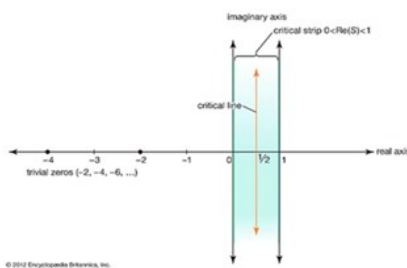
This milestone marks a turning point in mathematical history. Proof assistants have evolved from laborious manual translators into AI-accelerated reasoning engines capable of mastering high-level Grothendieck scheme theory.

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3.2 MATHEMATICIANS NARROW THE GAP ON THE RIEMANN HYPOTHESIS

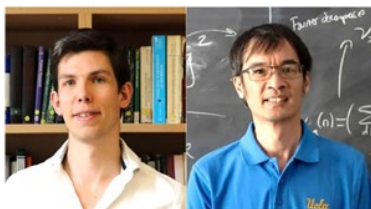
1. The Riemann Hypothesis, proposed by *Bernhard Riemann* in 1859, is widely seen as the ultimate expedition in the kingdom of pure mathematics. Riemann proposed that all non-trivial zeros of his zeta function, $\zeta(s) = \sum n^{-s}$, for $\text{Re}(s) > 1$ and extended by analytic continuation on $\mathbb{C} \setminus \{1\}$, lie precisely on a single vertical line in the complex plane: the 'critical line' where the real part of s is exactly $1/2$. Because the distribution of these zeros dictates the exact frequencies governing the prime numbers, proving this hypothesis would unlock the deepest structural secrets of arithmetic.



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While a complete proof remains unreachable, mathematicians have long measured progress by calculating the percentage of zeros that can be thoroughly proven to lie precisely on the critical line. In 1974, *Norman Levinson* proved that at least 34% of zeros lie on the line. In 1989, *Brian Conrey* broke the 40% barrier. For the next 37 years, however, that record moved at an unbearably slow pace, crawling up to 41.28% in 2011 and hitting what seemed like an impenetrable analytical ceiling.

On August 11, 2026, an international dream team of analytic number theorists led by Fields Medalist *James Maynard* (Oxford) (Left) and *Terence Tao* (UCLA) (Right) published a monumental 112-page paper that shattered the long-standing stalemate. The authors proved that at least 44.15% of all non-trivial zeros are strictly simple (meaning each zero has multiplicity exactly one) and lie directly on the critical line.



To achieve this spectacular advance, Maynard and Tao planned an entirely new class of 'multi-dimensional pseudoprime mollifiers' - mathematical reducing functions designed to cancel turbulent oscillations in the zeta function. By creating breakthrough techniques from spectral sieve theory and quantum chaos, they successfully controlled higher-order prime correlations, capturing an unusual fraction of zeros.

The paper delivered a second historic result regarding the De Bruijn - Newman constant, denoted by Λ , the constant arises from studying a one-parameter family of functions $H_t(z)$ (the de Bruijn-Newman functions) defined for a real parameter t and a complex variable z . They are expressed via the integral transform: $H_t(z) = \int_0^\infty e^{tu^2} \Phi(u) \cos(zu) du$, where $\Phi(u)$ is a super-exponentially decaying function constructed from the Riemann ξ function.

In 1950 De Bruijn proved that if $H_t(z)$ has only real zeros for some value t , it will also have only real zeros for any larger value $t' > t$. And in 1976 C. M. Newman proved the existence of a finite, unique real constant Λ such that $H_t(z)$ has only real zeros if and only if $t \geq \Lambda$.

The function $H_0(z)$ at $t = 0$ corresponds directly to the undeformed Riemann ξ function (whose real zeros correspond to the critical line zeros of the Riemann zeta function). If the Riemann Hypothesis is true, $H_0(z)$ has only real zeros, meaning Λ must be less than or equal to 0. Thus the Riemann Hypothesis is mathematically equivalent to the statement that $\Lambda \leq 0$.

In 2018, *Brad Rodgers* and *Terence Tao* proved that $\Lambda \geq 0$. In their August 2026 paper, Maynard and Tao established the new world-record upper bound: $0 \leq \Lambda < 0.000045$, squeezing the constant into the narrowest corridor in history.

2. In another development on the Riemann hypothesis, according to the blog of Mindstudio on August 13, 2026 [3], an unreleased Anthropic research model improved a specific zero-free region bound (i. e. unconditional domains in the critical strip ($0 < \text{Re}(s) < 1$) where $\zeta(s)$ is guaranteed to have no zeros) from 41.6% to 67.2%.

The model spun up 60 Claude sub-agents, generated 31 million output tokens, ran about 2,400 shell commands, and wrote hundreds of Python scripts during the process. the sub-agents tried roughly 650 approaches, before they found the one that did.

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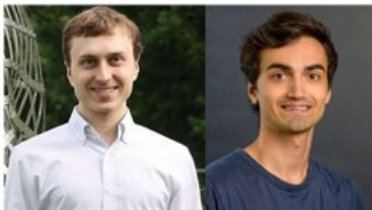
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3.3 A BREAKTHROUGH ACHIEVED ON GAUSS'S RIDDLE, UNSOLVED FOR 200 YEARS

In his 1801 magnum work *Disquisitiones Arithmeticae*, German mathematician *Carl Friedrich Gauss* wrote about a cyclical mystery. The puzzle involves primitive integral quadratic forms, such as $ax^2 + bxy + cy^2$, a, b, c as integers and $\gcd(a, b, c) = 1$. Setting the form equal to a number fixes it into an equation that can be plotted on an xy -graph.

Gauss described a method to combine two of these forms with coefficients $(a_i, b_i, c_i), i = 1, 2$, with same discriminant D and satisfying above conditions and the additional compatibility condition that $\gcd(a_1, a_2, \frac{b_1+b_2}{2}) = 1$, to produce a third form with coefficients (A, B, C) , where $A = a_1 \cdot a_2$; B is chosen such that: $B \equiv b_1 \pmod{2a_1}$; $B \equiv b_2 \pmod{2a_2}$, $B^2 \equiv D \pmod{4A}$; and $C = (B^2 - D)/4A$. He called the operation a “composition”.

Gauss defined an equivalence relation on primitive integral binary quadratic forms having fixed discriminant D . Two forms f and g are called equivalent if there exist integers α, β, γ and δ such that $f(\alpha x + \beta y, \gamma y + \delta y) = g(x, y)$, where $\alpha\delta - \beta\gamma = 1$. Gauss showed that equivalence classes of primitive binary quadratic forms having fixed discriminant D with above composition rule form a finite abelian group.



Using the composition, he combined a quadratic form, say Q , with itself to find a new form, Q^2 . Composing Q^2 with Q again, he got a third form, Q^3 . But as he kept repeating the steps over and over, he found that, after a finite number of iterations, the composition cycled through all possible distinct forms of the type Q^k and reset to the original form Q .

Gauss could see that the cycle resets regardless of his starting form, but he could not find any sort of rule governing the cycle's length. Until recently, nor could any other mathematicians.

Then, in 1983, mathematicians Henri Cohen and Hendrik Lenstra suggested a new approach using probability. Instead of trying to find the exact length of a single cycle, they created a formula

to predict the average - not the exact - length a cycle would take to reset. While their theory matched experimental data perfectly, nobody could prove that it was mathematically true.

The Cohen-Lenstra conjectured that the finite abelian groups do not appear with equal probability. Instead, they appear inversely proportional to the size of their automorphism group. This stood, generally accepted but unproven, for more than 40 years. Now Harvard University mathematician *Aaron Landesman* (Left) and Institute for Advanced Study Clay Research Fellow *Ishan Levy* (Right) have found a new framework that provides a major solution to the Cohen-Lenstra conjecture.

In 2012, a group of researchers thought they had found a proof by translating the numbers into geometric shapes, but a subtle error was later discovered, forcing them to retract it.

Landesman and Levy set out to fix that exact flaw. By using a field called homotopy theory, they built a new framework. Using this perspective, they proved that the complex geometric patterns eventually settle into a predictable, repeating state.

This breakthrough does more than just solve an old riddle. The new framework created by Landesman and Levy has already been used to unlock two other major, unrelated conjectures regarding number systems and modern encryption mathematics. Experts say this tool will likely become a new standard in the field, showing the incredible power of mixing different branches of mathematics to solve the world's oldest mysteries.

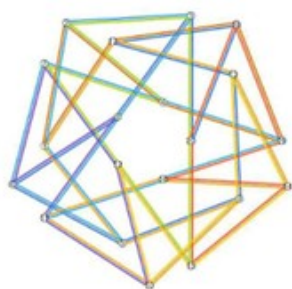
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3.4 AI CRACKS 50-YEAR-OLD CYCLE DOUBLE COVER CONJECTURE IN JUST ONE HOUR

In a major milestone for artificial intelligence in scientific research, on July 11, 2026, OpenAI officially announced that *GPT-5.6 Sol Ultra* has generated a complete proof of the Cycle Double Cover Conjecture - a legendary puzzle in graph theory that remained unsolved for over five decades. Remarkably, the AI completed the proof in less than an hour, well ahead of the eight-hour computational window researchers allocated for the task.

Formulated in the 1970s by several renowned mathematicians, the conjecture addresses networks of interconnected points and pathways (graphs): Consider an interconnected road network without dead ends or single bottleneck bridges - meaning closing any single road never cuts off one section of the network from the rest. The conjecture states that you can always lay out a set of circular routes so that every single road is traversed by exactly two routes - no more, no less.



For half a century, mathematicians have exhausted every possible effort to prove this conjecture. *Jaeger* proved that the conjecture holds for planar graphs; *Szekeres* proved it holds for 3-edge-colorable cubic graphs; *Alspach*, *Goddyn*, and *Zhang* proved it holds for bridgeless graphs with no Petersen minor. However, all these results came with additional constraints. The complete, unqualified affirmative proof had never been achieved - until the arrival of GPT-5.6 Sol Ultra.

OpenAI tackled the problem by deploying 64 independent AI agents running in parallel, governed by strict research rules: In the initial stage, agents were forced to explore wildly different angle - structural, algebraic, and network-flow approaches - without knowing which directions seemed most promising. A dedicated “correction squad” scrutinized every draft argument, actively chasing for counting mistakes, circular logic, or unverified claims. The system banned hand-waving shortcuts like declaring a step “obvious”, requiring verifiable constructions at every stage.

Rather than relying on brute-force computation, the AI executed an elegant, conceptual strategy: Simplification: It narrowed the challenge down to “bridgeless cubic graphs” (networks where every junction connects exactly three paths), borrowing a known principle that solving this core case

solves all graphs. This is because any graph can be topologically transformed to fall within the category of cubic graphs.

Assigning Labels: Using the previously proven result that every bridgeless graph nowhere-zero 8-flow, (i. e. there exists a function $\phi : E \rightarrow \mathbb{F}_2^3$, a three dimensional vector space over 2 elements field $\mathbb{F}_2$), that satisfies **Kirchhoff's Current Law**: at every vertex $v \in V$, Sum of the labels $\phi(e)$ for edges entering v = Sum of $\phi(e)$ for edges leaving v , and $\phi(e) \neq 0$ for all $e \in E$. The AI proposes a lemma: If every edge e can be assigned a set P_e containing two elements such that at every vertex, each element either appears 0 times or exactly 2 times - then the graph definitely has a cycle double cover.

The Algebraic Breakthrough: For proving the existence of such sets P_e , it converted the geometric network rules into a massive system of balance equations. By showing that this system of equations always balances out at every junction, the AI proved that a valid double cover must always exist [3].

Mathematicians have greeted the announcement with intense interest. Early reviewers noted that the AI's approach relies on surprisingly classical techniques that human researchers likely overlooked due to sheer complexity. Because the proof was written in standard mathematical prose rather than computer-verified formal code, graph theorists worldwide are now independently auditing the text to confirm every link in the AI's chain of logic.

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3.5 AI SOLVES 90-YEAR-OLD MYSTERY: DISPROVES THE JACOBIAN CONJECTURE

The Jacobian conjecture is a conjecture concerning polynomial functions from n -dimensional space to itself which was originally formulated in two dimensions by Ludwig Kraus in 1884. Later, the modern version of the Jacobian conjecture in n dimensions was formulated by Ott-Heinrich Keller in 1939 as follows:

Jacobian conjecture: Let $\mathbb{K}$ be a field of characteristics 0 and $F : \mathbb{K}^n \rightarrow \mathbb{K}^n$ be a polynomial function defined as $F(x_1, x_2, \dots, x_n) = (f_1(x_1, x_2, \dots, x_n), \dots, f_n(x_1, x_2, \dots, x_n))$ where $f_1, f_2, \dots, f_n$ are polynomial function of the variables $x_1, x_2, \dots, x_n$. Let J_H , the determinant of the $n \times n$ Jacobian matrix, be a non-zero constant then F has an inverse function $G : \mathbb{K}^n \rightarrow \mathbb{K}^n$ that is regular, meaning its components are polynomials.

The conjecture is number 16 in Stephen Smale's 1998 list of Mathematical Problems for the Next Century. Mathematicians have spent decades trying to prove the conjecture. That long-standing deadlock has now ended with the help of artificial intelligence.

Mathematician *Levent Alpöge*, a fellow at Harvard University affiliated with Anthropic, wrote on X on 19 July that the Jacobian conjecture is actually false for $n \geq 3$, giving a tiny, 216-character counterexample as proof. The polynomial map $F : \mathbb{C}^3 \rightarrow \mathbb{C}^3$, is given by $F(x, y, z) = ((1 + xy)^3 z + y^2(1 + xy)(4 + 3xy), y + 3x(1 + xy)^2 z + 3xy^2(4 + 3xy), 2x - 3x^2 y - x^3 z)$.

The Jacobian determinant of this function is:

$$\begin{vmatrix} 3yz(1+xy)^2 + 7y^3 + 6xy^4 & 3xz(1+xy)^2 + 8y + 21xy^2 + 12x^2y^3 & (1+xy)^3 \\ 3z(1+xy)^2 + 6xyz(1+xy) + 12y^2 + 18xy^3 & 1 + 6x^2z(1+xy) + 24xy + 27x^2y^2 & 3x(1+xy)^2 \\ 2 - 6xy - 3x^2z & -3x^2 & -x^3 \end{vmatrix}$$

It can be verified that the value of the determinant is -2 , a nonzero constant [3]. However, the map is not invertible, as it maps multiple distinct points to the same image. For example, Alpöge gives $F(0, 0, -1/4) = F(1, -3/2, 13/2) = F(-1, 3/2, 13/2) = (-1/4, 0, 0)$. This function can be easily generalised to give a counter example in case when $n \geq 4$. In this counter example, since all

coefficients and all three witness points are rational, so the same counterexample works over every field of characteristic zero. It is to be noted that the Jacobian problem is still open for $n = 2$. Finding this single flaw among complex, high-dimensional functions is an exhausting needle-in-a-haystack task for humans, but an ideal domain for high-speed algorithmic reasoning.

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3.6 AWARDS

3.6.1 Fields Medals



Every four years, the global mathematical fraternity gathers for the International Congress of Mathematicians (ICM) to celebrate the highest intellectual peaks of the discipline, and award the Fields Medals - often greeted as the Nobel Prize of Mathematics. Awarded to exceptional researchers under the age of 40, the 2026 laureates were officially announced during the opening ceremony in July 2026 in Helsinki.



On 23 July 2026, at the opening ceremony of the International Congress of Mathematicians in Philadelphia, the International Mathematical Union awarded the prestigious Fields Medal 2026, mathematics' highest honor for researchers under 40, to four rising stars of mathematics whose work spans kinetic theory, symplectic geometry, number theory, and harmonic analysis. Each winner received a gold medal bearing the face of Archimedes and 15,000 Canadian dollars, but the real prize, as always, is the recognition that their work has already reshaped the boundaries of what mathematics can explain.

Jacob Tsimerman: Known for his Landmark Proof of Griffiths Conjecture



Jacob Tsimerman, 38, of the University of Toronto, stands out as the first Fields Medalist to build his entire primary academic career in Canada. He was cited for extending o-minimality, a logical framework borrowed from model theory, into arithmetic and complex algebraic geometry, highlighted by his landmark proof of the Griffiths' conjecture on the algebraicity of period map images, which ties directly into the Hodge conjecture, one of the seven Millennium Prize Problems. Tsimerman's exceptional early academic trajectory includes winning gold medals at the International Mathematical Olympiad two years running before completing his doctorate at Princeton, cementing an uninterrupted life devoted to advanced arithmetic geometry.

John Pardon: Solved a Knot problem before graduating



John Pardon, 37, now at Stony Brook University in New York, was honored for a profound body of work in symplectic geometry and pseudo-holomorphic curves that has reshaped modern topology while deeply influencing representation theory and quantum physics. His academic excellence surfaced astonishingly early when he solved a major open conjecture concerning knots as a 21-year-old undergraduate, later graduating as valedictorian of Princeton's class of 2011. He subsequently earned his doctorate at Stanford. His foundational academic profile is further decorated by three consecutive gold medals

at the International Olympiad in Informatics from 2005 to 2007, followed by stellar contributions to geometric topology that earned him recognition as a leading global mathematical scholar.

It is a pleasure to mention here that Prof. Pardon was one of the invited speakers at the symposium on “Symplectic Topology”, held during the inaugural TMC Conference, organized in cooperation with the American Mathematical Society (AMS), at the Banaras Hindu University (BHU), Varanasi, during 14-17 December 2016. Congratulations to Prof. Pardon on behalf of The Mathematics Consortium (India), for his excellent achievement.

Yu Deng: Highlighted The Physics Hidden Inside Colliding Spheres



Yu Deng, 37, a professor at the University of Chicago, was recognized for his monumental work in partial differential equations (PDE). His achievements include the rigorous long-time derivation of the Boltzmann equation, one of the central pillars in kinetic theory and fluid dynamics, directly from the microscopic mathematics of colliding hard spheres. He also derived wave kinetic equations from nonlinear diffusing systems and formulated probabilistic approaches to nonlinear Schrödinger dynamics. Adding to his academic honors, Deng received a Sloan Research Fellowship in 2021, the MCA Prize in 2025, and the prestigious Clay Research Award alongside the 2026 Leonard Eisenbud Prize for Mathematics and Physics. Deng’s path began in a Chinese high school classroom, where a gold medal at the International Mathematical Olympiad earned him direct admission to Peking University, leading to his doctorate at Princeton.

Hong Wang: Breaking boundaries in Geometric Measure Theory



Hong Wang, 35, of NYU’s Courant Institute and France’s Institut des Hautes Études Scientifiques, became only the third woman in the Fields Medal’s 90-year history to win the Fields Medal, following *Maryam Mirzakhani* in 2014 and *Maryna Viazovska* in 2022. Displaying exceptional academic aptitude early on, she skipped primary grades and excelled on China’s national college entrance exam at 16, before committing entirely to mathematics. In February 2025, alongside collaborator *Joshua Zahl*, she posted a 127-page proof resolving the three-dimensional *Keakeya conjecture*, a question first posed in 1917 asking the smallest possible space a needle can sweep while rotating to point in every direction, a problem so resistant to proof that *Terence Tao* once called it one of the most sought-after open problems in geometric measure theory. Her broader academic citation recognizes her deep, transformative advances in Fourier restriction, Falconer distance sets, and planar Furstenberg sets.

3.7 SHAYAN OVEIS GHARAN WINS THE 2026 IMU ABACUS MEDAL



Theoretical computer scientist *Shayan Oveis Gharan* of the University of Washington has received the prestigious 2026 IMU Abacus Medal. This award is one of the highest global honors for researchers under the age of 40 in information sciences. International Mathematical Union praised his pioneering work. He connects theoretical computer science with algebraic geometry, spectral theory, and the geometry of polynomials and thereby bridging Algorithms and Algebra. His major achievements include:

The Traveling Salesperson Problem: He shattered a 50-year-old barrier in approximation algorithms for the metric version of this famous route-planning puzzle.

Markov Chain Monte Carlo: He resolved longstanding sampling challenges, including the famous Mihail-Vazirani conjecture.

New Mathematical Insights: His advanced techniques explain how modern algorithms navigate massive computational spaces and complex networks.

These breakthroughs fundamentally change how scientists understand the power and limits of computation in our digital world.

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3.8 OBITUARY

3.8.1 Queen of Modern Partial Differential Equations - Prof. Nina Nikolaevna Uraltseva Died at the Age of 92



Prof. Nina Nikolaevna Uraltseva, universally acclaimed as the “Matriarch of Modern Partial Differential Equations”, passed away in St. Petersburg, Russia, on July 15, 2026, at the age of 92. Over a tireless seven-decade career, her research fundamentally reshaped non-linear analysis, variational calculus, and geometric measure theory.

Born in Leningrad (now St. Petersburg), Nina Uraltseva survived the tragic, brutal Siege of Leningrad as a young child during World War II. Demonstrating a prodigious mathematical intellect, she entered Leningrad State University, graduating in 1956. She formed an iconic intellectual partnership with the legendary *Olga Aleksandrovna Ladyzhenskaya*. Together, Ladyzhenskaya and Uraltseva tackled Hilbert’s Nineteenth and Twentieth Problems, seeking to determine whether solutions to classical non-linear variational problems are necessarily smooth. Their 1964 monumental masterpiece, ‘Linear and Quasilinear Equations of Elliptic Type’ (translated into English in 1968), became the bible of partial differential equations. In this work, Uraltseva formulated the foundational Harnack-type inequalities, a priori Hölder estimates, and boundary regularity theories for quasilinear equations that are taught to every graduate student in analysis today.

Over her legendary career, Prof. Uraltseva served for decades as the Head of the Department of Mathematical Physics at St. Petersburg State University, mentoring hundreds of mathematicians. She was awarded the Chebyshev Prize of the USSR Academy of Sciences, the State Prize of the Russian Federation, and the Humboldt Research Award. Her profound theorems remain the bedrock upon which modern fluid mechanics, minimal surface theory, and general relativity are built.

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□ □ □

4. Problem Corner

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In the July 2026 issue of TMC Bulletin, we posed two problems, One from Number Theory and another from Analysis. So far no solution for any of these problems has been received from any of our readers. We give below the solution to these problems provided by the respective problem proposers. Also, in this issue we pose two problems, one from Number Theory and one from Analysis for our readers. **Readers are invited to email their solutions to Dr. Udayan Prajapati (ganit.spardha@gmail.com), Coordinator, Problem Corner, before 25th November, 2026.** Selected innovative solutions will be published in a subsequent issue of the Bulletin.

Problem from Number Theory (Posed by K. B. Subramaniam from Bhopal and Amarnath Murthy from Ahmedabad): Prove that there are infinitely many perfect squares with the property that the sum of their digits is also a perfect square.

Solution: Theorem: The sum of the digits of the square of the number “one followed by n threes” is a perfect square for all $n = 60t^2 + 76t + 24$, for any value of t and is equal to $(30t + 19)^2$.

Proof: Consider a sequence 13, 133, 1333, 13333 .. whose general term T_n is a number ‘one followed by n threes’. One can easily verify that the sequence of squares of numbers in the above sequence is:

$$169, 17689, 1776889, 177768889, \dots$$

It is evident that this sequence of squares follows a pattern. We formally prove this in the following **Proposition I:** T_n^2 is the number S_n = one followed by $(n - 1)$ seven's, followed by a six, followed by $(n - 1)$ eight's followed by a nine”.

Proof: The general term of the previous root sequence is $T_n = 10^n + 3(11 \dots 1)(n \text{ times } 1)$

That is, $T_n = 10^n + 3 * (10^n - 1)/9 = (1/3)(4 * 10^n - 1)$

Therefore, $T_n^2 = (1/9)(16 * 10^{2n} - 8 * 10^n + 1)$.

The general term of the square sequence is $S_n = 1 \text{ 777 ... 6 888 ... 9 (7 and 8 repeat } (n - 1) \text{ times)}$

$$= 10^{2n} + 7 * 10^{n+1}(111 \dots (n - 1) \text{ times } 1) + 6 * 10^n + 80 * (111 \dots (n - 1) \text{ times } 1) + 9$$

$$= 10^{2n} + 7 * 10^{n+1} * (10^{n-1} - 1)/9 + 6 * 10^n + 80(10^{n-1})/9 + 9$$

$$= (1 + 7/9)10^{2n} + (-70/9 + 6 + 8/9)10^n + (-80/9 + 9)$$

$$= (1/9)(16 * 10^{2n} - 8 * 10^n + 1) = T_n^2.$$

This completes the proof of Proposition I.

The sum of the digits of this type of square number is $1 + 6 + 9 + (n - 1)(7 + 8) = 15n + 1$.

The only thing that remains is, to prove that the Diophantine equation $15n + 1 = k^2$ has infinitely many solutions.

Proposition II: If $n = 60t^2 + 76t + 24$ then $15n + 1$ is a perfect square for all values of t .

Proof:

$$15n + 1 = 15(60t^2 + 76t + 24) + 1$$

$$= 900t^2 + 2 \times 30 \times 19t + 361 = (30t + 19)^2 \text{ which is a complete square.}$$

Together with Proposition I, the theorem stands proved and in turn solves the Problem I.

Problem from Analysis (Posed by Proposer, Udayan Prajapati from Ahmedabad): Prove that every real number can be expressed as $x = \pm c_1 \pm c_2 \pm c_3 \pm \dots \pm c_n$ for some positive integer n and some elements $c_1, c_2, c_3 \dots, c_n$ of the Cantor set C .

Solution: It is known that $C + C = [0, 2]$ and 0, 1 are the element of the Cantor set*.

It is enough to prove the result for $x > 0$.

Let n be the smallest positive integer such that $n - 2 < x \leq n$. That is $0 < x - n + 2 \leq 2$. Thus, there are two numbers c_1 and c_2 from the cantor set such that $x - n + 2 = c_1 + c_2$.

Hence $x = c_1 + c_2 + n - 2 = c_1 + c_2 + 1 + \dots + 1$.

*<https://math.stackexchange.com/questions/4155900/prove-that-cc-0-2-where-c-is-the-cantor-set>

Problems for this issue

Problem 1: (Posed by Vinay Acharya, Fergusson College, Pune): Represent the number $989 \times 1001 \times 1007 + 320$ as the product of primes.

Problem 2: (Posed by B. Sury, retired from ISI, Bangalore): Find all continuous functions from $[0, 1]$ to itself such that $f(f(f(x))) = x$ for all x .

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5. International Calendar of Mathematics Events

Ramesh Kasilingam

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December 2026

- December 7-11, 2026, Computational Methods and Function Theory (CMFT 2026): International Conference, Indian Institute of Technology Madras, Chennai, Tamil Nadu, India. <https://ge.iitm.ac.in/cmft-2026>
- December 9-11, 2026, The International Conference on Mathematics, Statistics and Their Applications (ICMSA 2026), Information Technology Service Centre (ITSC), Chiang Mai University, Thailand. https://iasc-ars2026.icdi.cmu.ac.th/ICMSA_IMT
- December 14-18, 2026, Workshop on Euler Systems and p-adic L-functions, University of Santiago de Compostela, Galicia, Spain. <https://sites.google.com/view/workshop-on-euler-systems-and-/>
- December 14-16, 2026, MathConnect2026 - 2nd International Conference on Mathematics and its Applications, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia. <https://mathconnect26.com/en>

January 2027

- January 4-8, 2027, Mathematical Analysis and Modelling of Maxwell's Equations in Complex Media, ICERM, Brown University, Providence, Rhode Island. https://icerm.brown.edu/program/topical_workshop/tw-27-mam
- January 4-9, 2027, 2027 LiBERA Winter School on Stochastic Interacting Systems Kilpisjarvi Biological Station, Kilpisjarvi, Finland. <https://math.aalto.fi/en/research/stochastics/events/SIS2027/>
- January 11-15, 2027, Advances and Challenges in Computational Partial Differential Equations, ICERM, Brown University, Providence, Rhode Island. https://icerm.brown.edu/program/topical_workshop/tw-27-acc
- January 12-15, 2027, Joint Mathematics Meeting (JMM) McCormick Place, Lakeside Centre and South Building, Chicago, IL. <https://www.ams.org/meetings/national/national>
- January 19-22, 2027, Revisiting Fundamental Problems Workshop: Old Problems in Irrationality, 17 Gauss Way, Berkeley, CA 94720. <https://www.slmath.org/workshops/1161>
- January 20 - April 27, 2027, Metric Algebraic Geometry: Going Global, ICERM, Brown University, Providence, RI. https://icerm.brown.edu/program/semester_program/sp-s27

- January 24-25, 2027, SIAM Symposium on Algorithm Engineering and Experiments (ALENEX27), Sheraton Philadelphia Downtown, Philadelphia, Pennsylvania, U.S.
<https://www.siam.org/conferences-events/siam-conferences/alenex27/>
- January 24-27, 2027, ACM-SIAM Symposium on Discrete Algorithms (SODA27), Sheraton Philadelphia Downtown, Philadelphia, Pennsylvania, U.S.
<https://www.siam.org/conferences-events/siam-conferences/soda27/>
- January 25-26, 2027, SIAM Symposium on Simplicity in Algorithms (SOSA27), Sheraton Philadelphia Downtown, Philadelphia, Pennsylvania, U.S.
<https://www.siam.org/conferences-events/siam-conferences/sosa27/>

February 2027

- February 3-5, 2027, 2nd International Conference on Recent Advances in Mathematics and its Applications (ICRAMA-2027), Department of Mathematics, Tripura University, India.
<https://sites.google.com/tripurauniv.ac.in/icrama-2027>
- February 19-21, 2027, 2nd International Conference on Mathematical Optimization Theory and Applications (ICMOTA 2027), Indian Institute of Technology (BHU), Varanasi, India.
<https://conferences.iitbhu.ac.in/ICMOTA-2027/>
- February 22-24, 2027, SIAM Conference on Applied and Computational Discrete Algorithms (ACDA27), The David L. Lawrence Convention Centre, Pittsburgh, Pennsylvania, U.S.
<https://www.siam.org/conferences-events/siam-conferences/acda27/>
- February 22-26, 2027, Hot Topics Workshop: Geometric Sen Theory, The p-adic Simpson Correspondence, and Modularity, 17 Gauss Way, Berkeley, California, USA.
<https://www.slmath.org/workshops/1165>
- February 23-26, 2027, 12th UFPB Summer Conference in Mathematics (SCM 2027), Department of Mathematics, Universidade Federal Da Paraíba (PPGMat-UFPB), Brazil.
<https://sites.google.com/academico.ufpb.br/scm2027/>

March 2027

- March 12-14, 2027, The 43rd Southeastern Analysis Meeting (SEAM), George Mason University, Fairfax, Virginia, USA. <https://seam-43.github.io/>
- March 15-19, 2027, Integral Geometry of Algebraic Varieties, ICERM, Brown University, Providence, Rhode Island.
https://icerm.brown.edu/program/semester_program_workshop/sp-s27-w2
- March 15-19, 2027, Number Theory and Representation Theory: The Legacy of Benedict Gross, and Beyond, Harvard University Science Centre, Cambridge, Massachusetts, USA.
<https://www.math.harvard.edu/event/benedict-gross-conference/>
- March 18-20, 2027, Spring Topology and Dynamics Conference, Columbus, Ohio, USA.
<http://stdc60.org/>
- March 20-21, 2027, 2027 AMS Spring Southeastern Sectional Meeting, University of Alabama - Birmingham, Birmingham, Alabama, USA.
https://www.ams.org/meetings/sectional/2341_program.html

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Team India wins 2 Gold and 4 Silver Medals at 67th International Mathematical Olympiad (IMO)

The 67th International Mathematical Olympiad was hosted in Shanghai, China from July 10 to July 21, 2026. The core competition consisting of two 4.5-hour papers was held on July 15 and 16. 666 students from 117 countries participated in the event, making it the largest IMO participation in the history of IMO, which began in 1959. India won two gold and four silver medals at the event. The result gave the team an unofficial seventh place, making it the fourth consecutive top-10 finish by Indian teams. Team members and their performance is given below



Contestant L-R ↓	Rank	%, Points	Award
2. Mr. Bairav Murugan (Ka)	105	84.4 25	G
3. Mr. Aarav Gupta (Delhi)	34	95.1 30	G
4. Mr. Abel George Mathew (Ka)	17	97.6 35	S
5. Mr. Kanav Talwar (Delhi)	76	88.7 27	S
6. Ms. Shreya Shantanu Mundhada (Mumbai)	117	82.6 24	S
7. Ms. Sanjana Philo Chacko (Ke)	105	84.4 25	S

1. Leader: **Anant Mudgal**, CEO, Mathelots Pvt. Ltd.

8. Deputy Leader: **Mainak Ghosh**, HBCSE, Mumbai.

Team MMC India Shines at International Mathematics Competition (IMC)

The International Mathematics Competition for University Students 2026 was held in Blagoevgrad, Bulgaria, from 27th July to 3rd August 2026. Hosted by the American University in Bulgaria and organized by University College London, the competition brought together 447 of the world's finest undergraduate students, representing 80 university teams from across the globe. Representing India through the **Madhava Mathematics Competition (MMC)**, organized by Bhaskaracharya Pratishthana, Pune, Team MMC India delivered an exceptional performance. It may be recalled that this is just the second participation of a Team MMC at the IMC. Its success has brought international recognition to Indian undergraduate mathematics. Team members and their performance is given below:



Contestant L-R ↓	Prize	Points
2. Nirav Bhattad (IIT Bombay)	Second	79
3. Parth Vartak (IIT Bombay)	First	65
4. Bhavya Tiwari (IIT Bombay)	Grand First	57
5. Shounak Kar (ISI Kolkata)	Grand First	57

Team MMC India Led by

1. **Dr. Vinaykumar Acharya**, Fergusson College, Pune.

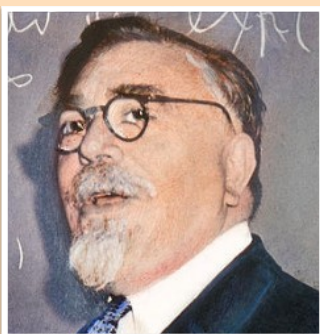
6. **Dr. Udayan Prajapati**, St. Xavier's College, Ahmedabad.

We congratulate all the contestants of IMO 2026 and IMC 2026 for their excellent performance at the international level. We also very much appreciate the dedication and collective efforts of all the team leaders, mentors, regional coordinators of the Madhava Mathematics Competition which made this historic success possible.



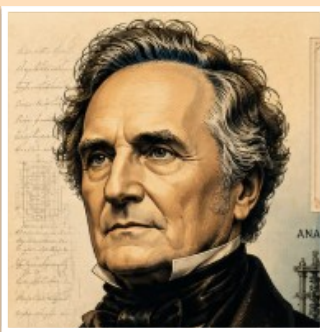
Edmond Halley (29 Oct. 1656 - 14 Jan. 1742)

An English mathematician, astronomer, physicist. Constructed an observatory on Saint Helena. Recorded a transit of Mercury across the Sun. Aided in observationally proving Newton's laws of motion, and used them to compute the periodicity of a Comet which was named after him, after its predicted return in 1705. Expanded contemporary star maps. Made observations on the condition of terrestrial magnetism.



Norbert Wiener (26 Nov. 1894 - 18 March 1964)

An American mathematician, computer scientist. A child prodigy. Worked on Electronic communication, and control systems. Credited as being one of the first to theorize that all intelligent behavior was the result of feedback mechanisms, that could possibly be simulated by machines. Helped develop the theories of cybernetics, robotics, computer control, and automation.



Charles Babbage (26 Dec. 1791 - 18 Oct. 1871)

An English mathematician, philosopher and mechanical engineer. Originated the concept of a digital programmable computer. Invented the first mechanical computer, known as Babbage's Analytical Engine. Also designed the first computer printers. Worked on the electrodynamics of Arago's rotations with Herschel. Elected as a Fellow of the Royal Society in 1816.

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