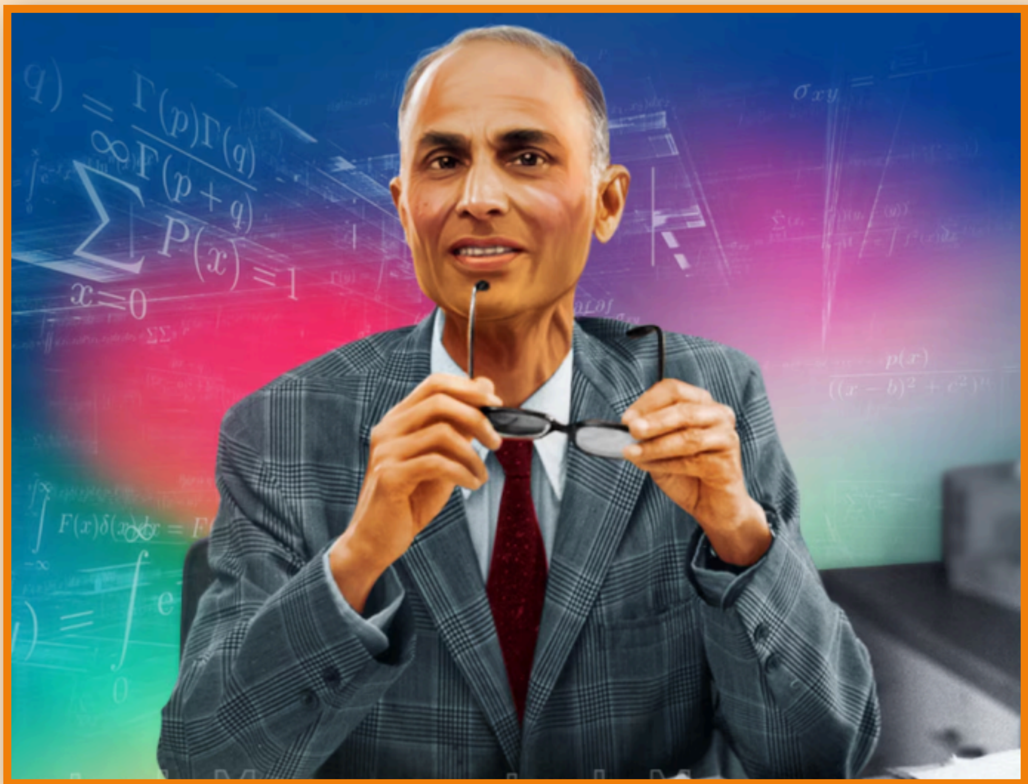
The Mathematics Consortium

BULLETIN

October 2024 A TMC Publication Vol. 6, Issue 2

Dedicated to the memory of

Harish-Chandra



a great mathematician of Indian origin,
on the occasion of his 101st birth anniversary

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Contents

About the Cover Page: *The front cover image is of Harish-Chandra who, as noted by Prof. M. S. Raghunathan in his article, is one of the great names in twentieth century mathematics, the greatest of Indian origin after Srinivasa Ramanujan.*

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From the Editors' Desk

There has been tremendous advancement in recent years in the field of Artificial Intelligence (AI). AI systems like, Generative Pre-trained Transformer 4 (GPT 4), BERT are capable of generating human-quality text, translating languages, writing different kinds of creative content, and answering your questions informatively. Google's DeepMinds AlphaTensor is capable of finding faster Matrix multiplication algorithms; and more recently developed, AlphaProof and AlphaGeometry 2 can solve problems in Geometry as well as complex mathematical problems posed in various Olympiads. All these developments are rapidly transforming various fields, and education is no exception. The potential of AI to revolutionize how we teach and learn is immense.

A key question that arises here is, whether this scenario will have significant impact on Teachers' role in schools and colleges? While some experts have expressed worries about AI replacing teachers, most experts believe that it will augment the role they play, freeing them from routine administrative tasks, to focus on more meaningful interaction with the students. Kai-Fu Lee, a prominent AI expert, has emphasized the importance of human-AI collaboration in education. Lee believes that teachers' unique skills in empathy, emotional intelligence, and critical thinking are irreplaceable. AI could be used to provide students with access to vast amounts of information and resources, and a teacher can play a crucial role in helping them to filter this information and improve their knowledge of the topics being taught. Lee emphasizes the need for teachers to be trained to effectively use AI tools, to evaluate AI algorithms, understand their limitations, and integrate them into their teaching practices.

This issue of TMCB we are dedicating to the memory of Harish-Chandra, on the occasion of his 101^{st} birth anniversary, celebrating the legacy of the great mathematician of Indian origin, who has left deep imprints on the mathematical world. In the opening article Prof. M. S. Raghunathan, outlines his biography, highlights his significant contributions to mathematics in general, and to Representation theory in particular. His reminiscences with Harish-Chandra and his family are recounted, quoting some interesting anecdotes.

In Article 2, Prof. M. H. Vasavada has expressed his concerns regarding mathematics education in Schools. He has also given very vital suggestions for motivating students and making teaching of mathematics interesting. Recent AI tools mentioned above, can indeed be used advantageously to implement these suggestions.

In Article 3, Dr. D. V. Shah gives an account of significant developments in the Mathematical world during recent past, including the current status of Geometric Langlands Conjecture, Riemann Hypothesis and development of new AI Systems. He also presents some highlights of the work of awardees of the 2024 Wolf Prize in Mathematics.

In Article 4, we pay our tributes to Indian mathematicians Prof. E. Samathkumar, a well-known graph theorist and an Executive Committee member of TMC, Prof. R. C. Gupta, a legendary Indian historian of mathematics, and Prof. J. N. Salunke, a dedicated teacher actively involved with TMC, who passed away recently.

Beginning with this issue we will be carrying a column, "A Peep into History of Mathematics". Under this column, Prof. S. G. Dani gives brief reviews of some recent papers in the area of history of mathematics.

Article 6 contains a review By Prof. Sukavanam of a book on Linear Algebra by Raju K. George and Abhijit Ajaykumar. In the Problem Corner, Dr. Udayan Prajapati presents 3 solutions to one of the problems posed in the last issue and a solution by late Prof. J. N. Salunke to a problem posed in October, 2023 issue. A new problem is posed for our readers. Dr. Ramesh Kasilingam gives a calendar of academic events, planned during January to May, 2025, in Article 8.

We are happy to bring out this second issue of Volume 6 in October, 2024. We thank all the authors, all the editors, our designers Mrs. Prajakta Holkar and Dr. R. D. Holkar, and all those who have directly or indirectly helped us in bringing out this issue on time.

Chief Editor TMC Bulletin

1. Harish-Chandra - Some Reminiscences

M. S. Raghunathan
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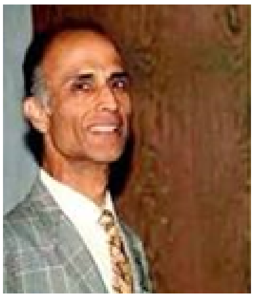


Harish Chandra

Harish-Chandra, as is well known, is one of the great names in twentieth century mathematics, the greatest of Indian origin after Srinivasa Ramanujan. I had the privilege of knowing him, but cannot claim to have been close to him; yet I had some memorable interactions with him as well as his wife Lalitha Harish-Chandra. I was also involved in some activities relating to Harish-Chandra. In this article I reminisce about some of these. I hope that the mathematical community will find it interesting. The photograph on the left was taken when Harish-Chandra was in his mid-fifties. When I first met him he looked very much like this except that he had not greyed at the temples.

I will first outline a biography of Harish-Chandra before I set out to talk of my reminiscences.

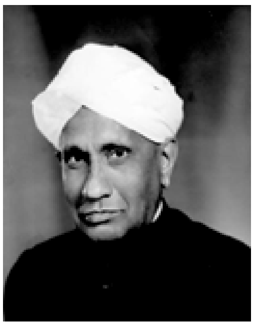
K. S. Krishnan

I will first outline a biography of Harish-Chandra before I set out to talk of my reminiscences. Harish-Chandra was born on October 11, 1923 in Kanpur. His father Chandrakishore, was an Engineer in the provincial service involved with irrigation in the United Provinces of British India and that meant frequent transfers. So Harish-Chandra spent much of his childhood divided between wherever his father was stationed and Kanpur where his maternal grandfather Shri Ram Sanehi Seth lived. The latter part of his schooling was in Kanpur. He learnt music and dance, the latter somewhat unusual for a boy. He retained a love of music throughout his life. After school Harish-Chandra went to Allahabad (now Prayagraj), one of the country's leading academic centres in those days, for his under-graduate studies. He studied Physics at Allahabad and was a brilliant student.

Homi J Bhabha

Harish-Chandra was born on October 11, 1923 in Kanpur. After a B.Sc. he went on to obtain a Masters in 1943. The eminent physicist K. S. Krishnan was one of his teachers. Harish-Chandra was fond of him and retained his affection and respect for him life-long. He passed his M.Sc. exam with flying colours. In one of the papers, he scored 100 percent: the examiner was C. V. Raman, a hard task-master, if ever there was one! At the university he took a course in French. It was taught by a Polish lady, Mrs. H. Kale, who was to become his mother-in-law later.

Harish-Chandra wanted to pursue research in Physics and Krishnan suggested that he may go to the Indian Institute of Science in Bangalore to work with H. J. Bhabha, then a professor there. With Krishnan's recommendation, Bhabha was happy to take him as a student and Harish-Chandra took up residence as a Paying Guest with the Kales; Professor Kale was the Librarian at IISc; his French teacher in Allahabad was Professor Kale's wife.

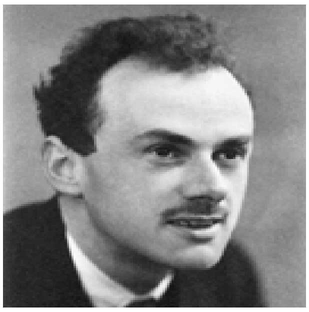


C. V. Raman

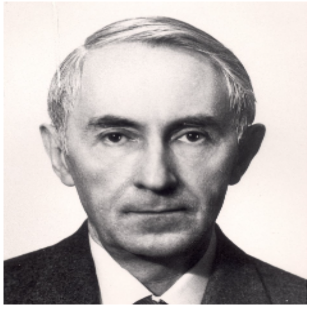
During his stay in Bangalore he got to know C. V. Raman well. He went on long walks with Raman. The great Physicist apparently enjoyed the company of the young man. In Bangalore, during 1943-45, Harish-Chandra wrote a few joint papers with Bhabha and some on his own. He then moved to Cambridge with Bhaba's help to work with the Nobel Laureate, P. A. M. Dirac.

Though he was technically attached to Dirac his meetings with him were far and few between. In Cambridge at a talk by the formidable Wolfgang Pauli he had the temerity to point out an error, an act which attracted considerable attention. Later during a visit to Zurich he became friend with Pauli. Dirac accepted an invitation to visit the Institute for advanced Study in Princeton during 1947-48. Harish-Chandra went with Dirac as his Assistant to the institute.

Harish-Chandra's interest in representations of the Lorentz group led him to the study of representations of semi-simple Lie groups in general. Claude Chevalley, a great mathematician and an expert on Lie groups was visiting at the institute at that time, and Harish-Chandra got in touch with him.



P. A. M. Dirac



Claude Chevalley

Contact with Chevalley essentially resulted in his becoming a full time mathematician. He found mathematics to be more exciting than Physics. In those days representation theory of semi-simple Lie groups was only of peripheral interest to the mathematical community at large. There were a few, mostly in Soviet Russia, who were pursuing the subject. Over the next decade and a half Harish-Chandra's path-breaking beautiful work brought it to the centre-stage in mathematics.

G. D. Mostow

During his stay at the institute, he formed a life- long abiding friend-ship with the American mathematician George Daniel Mostow, an expert in geometric aspects of Lie theory. Harish-Chandra spent the academic year 1949-1950 at Harvard University taking the opportunity to fill some of the lacunae in his mathematical knowledge. His talent came to the fore when he provided an elegant solution to a problem in number theory posed by Nagel in a graduate course he was teaching; a solution had eluded Nagel himself and the many others who were taking the course.

In 1950, he joined Columbia University faculty and was to remain there till 1963. The stay was interspersed with several visits abroad on leave. He spent the year 1952-53 as a Visiting Professor at the Tata Institute of Fundamental Research (TIFR), Bombay (now Mumbai). Bhabha (then director of TIFR) perhaps had ideas of getting Harish-Chandra back to India permanently. At that time, the Mathematics Faculty at TIFR consisted of D. D. Kosambi who had in fact moved to research in history, K. Chandrasekharan and K. G. Ramanathan both of whom were far removed in their mathematical interests from those of Harish-Chandra. Bhabha's extravagant life style (which he knew of, from his Bangalore days) perhaps put him off. Harish-Chandra gave up the idea of returning to India after this visit (if he ever entertained it at all).

The photographs of Lalitha Kale and Harish-Chandra above on the left perhaps date back to 1952. At right is a photograph of the married couple taken in Allahabad in January 1953 after the wedding. The wedding took place in Mysore (the Kales had moved there) in December 1952 in a fancy Hotel. Dan Mostow, wrote that the handsome Harish-Chandra and his beautiful bride were the cynosure of all eyes at the International Congress of Mathematicians (ICM) held in Amsterdam in 1954.

Starting in Princeton, Harish-Chandra's creativity resulted in a series of profound and original papers; the earlier ones among them appeared in the Transactions of the American Mathematical Society (TAMS). One of them won him the prestigious Cole Prize. They culminated in a theorem that described for a semi-simple Lie group G the decomposition of $L^2(G)$ into a direct integral of irreducible unitary representations and determining the Plancherel Measure. The early papers secured for him a tenured position at Columbia University, NewYork. During the years he spent there he gave, among others, a course on Class Field Theory, a subject somewhat far removed from his specialization, representation theory. Harish-Chandra's series of papers on infinite dimensional representations theory of Lie groups had many original ideas and made the once obscure subject accessible. His work was beautiful; and beauty of a work is certainly a characteristic that drags the work to centre-stage in mathematics. Harish-Chandra had done it single handedly - there were some mathematicians of high calibre, most of whom located in Soviet Russia, who were into representation theory. But none could match the strides he was making in the area.

Andre Weil

During his years in Columbia he took a year off to visit Paris. He enjoyed the Paris visit immensely and it was great for him professionally. At the end of his leave he wrote to Columbia asking for an extension of his leave by a year. The dean at Columbia wrote back a stiff letter refusing further leave. A somewhat disappointed Harish-Chandra consulted the great Andre Weil (who was also in Paris then and had become a good friend) about how he should respond. Weil told him that he was too good a mathematician for Columbia to afford losing him; and that he should not only insist on being given the extension but ask for a hike in his salary if Columbia wanted to retain him. Harish-Chandra took the advice; Columbia capitulated!

In 1963, Harish-Chandra was invited to become a Permanent Professor at the Institute for Advanced Study. That was indeed the highest possible recognition for mathematical achievement. He was the first Indian mathematician to be extended the honour. His work in the representation theory of semi-simple Lie groups is profoundly original and beautiful and could hold its own in the midst of the formidable achievements of the men he was joining. However there were among his colleagues men (Weil for example) with even greater achievements and on a broader front. He apparently worked at home and put in an appearance at the institute for attending or giving lectures or for the afternoon Tea. He met people only by appointment.

Fuld Hall of the IAS

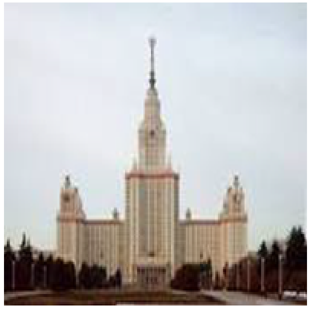
Beside is a photograph of the main building called Fuld Hall of The Institute for Advanced Study in Princeton, New Jersey, USA. This was the first building to come up. The building houses the library, a lecture hall, offices of some administrators and permanent professors as well as a lounge where Tea was served every afternoon. Harish-Chandra's office as well as those of most permanent professors was in an Annexe. Another permanent member at that time was Armand Borel, with whom Harish-Chandra wrote a joint paper - that is the only joint paper that Harish-Chandra ever wrote. The Borels and Harish-Chandras were neighbours and the families were very close indeed.

Congress of Mathematicians (ICM) held in Moscow

In this ICM a good number of mathematicians of stature led by I. M. Gelfand, working in the area of representation theory were present. For many years Moscow did not accord him the primacy in the area that was acknowledged everywhere else. But at this ICM in Moscow, Muscovites too fell in line - he was lionized, giving him immense pleasure. The venue of the ICM was Moscow State University. The invitation to give a plenary address was the highest recognition for his mathematical work that he received other than the permanent membership of the Institute for Advanced Study. For a man of his achievements, he received a rather small number of awards and honours.

Armand Borel

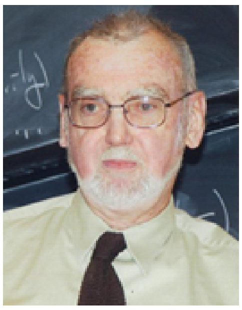
I. M. Gelfand



Moscow State University

By 1963, he had brought his work on representation theory to a culmination. However he continued to work with the same intensity as ever despite setbacks to his health. He turned his attention to the theory of representations of p-adic Lie groups and came up with profound contributions to it. After the early fifties his visits to India were far and few between. His last visit to India was in 1972. On his return from India he suffered a second heart attack after which he stopped visiting the country as he lacked confidence in the available health-care.

Harish-Chandra passed away a week after his 60th birthday in 1983. I was in Princeton when it happened. In the afternoon of that day the Harish-Chandras hosted a lunch at which I was present. Harish-Chandra was at his lively best at the lunch. He was, later, in the late afternoon, found lying dead on the road - he had gone out on a walk never to return. He had apparently died of a heart attack, his fourth. I was given the news on the telephone at mid-night by Armand Borel who later drove me to Harish-Chandra's house for me to pay my last respects. That then is a brief biography of Harish-Chandra.



Robert Langlands

Robert Langlands has written an Obituary Notice about Harish-Chandra soon after his death for the Royal Society. He gives an excellent short account of Harish-Chandra's life giving lot more details than what I have done above. Langlands also tells us something about Harish-Chandra's work. Langlands has himself done some deep work in representation theory and he is among the few who have an in-depth understanding of Harish-Chandra's work. Langlands admired Harish-Chandra greatly, an admiration that was fully reciprocated.

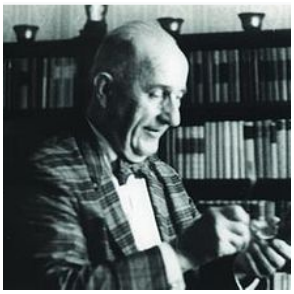
In 1984 Springer Verlag brought out 4 volumes on Collected Works of Harish-Chandra edited by a close friend and admirer V. S. Varadarajan. Much later in 2017, Springer published a collection of his posthumous papers with Ramesh Gangolli, also a friend and admirer of Harish-Chandra, and Varadarajan as joint editors.

I now embark on my reminiscences relating to Harish-Chandra. I first heard of Harish-Chandra soon after I joined TIFR as a student in 1960. His work, I learnt was in an area known as representation theory and that he had established himself as a leader in the field but there was no one at TIFR working in that area. I was told that in ISI Kolkata there were some people who had embarked on studying Harish-Chandra's work. One of them, V. S. Varadarajan, later became a leading expert in the field. Much later, in the seventies, there were some outstanding contributions to representation theory from M. S. Narasimhan, R. Parthasarathy and S. Kumaresan at TIFR.

Harish-Chandra was a candidate for the Fields Medal in 1958 but was not awarded the medal.

K. F. Roth

Rene Thom



C. L. Siegel

K. Chandrasekharan

That year's medals went to Rene Thom, a French topologist and Klaus Friedrich Roth, a German born Number Theorist working in England. Roth had settled a question that Siegel himself was interested in but had not succeeded in answering. Thom was arguably the most original of the topologists of the twentieth century. So the case for their being awarded the medals was quite strong. However, in the opinion of many mathematicians Harish-Chandra was a stronger candidate than Roth. There was a rumour current in TIFR that C. L. Siegel, the Chair of the Fields Medal Committee, was averse to giving the medal to Harish-Chandra as he was set against a second follower of the Bourbakis getting the medal. The first one in his reckoning being Thom. It was well known that Siegel was totally unsympathetic to the approach of the Bourbakis to mathematics. Thom, though a student of Henri Cartan, a founder of Bourbaki was far from being a follower of Bourbaki; nor could Harish-Chandra be considered a true follower. The Fields Medal Committee had as a member an Indian mathematician, K. Chandrasekharan (I learnt much later that Harish-Chandra was under the impression that Chandrasekharan, a fellow Indian, had not supported him and was cut up about it). Archival material now available discredits the rumour. Also Harish-Chandra's impression of Chandrasekharan's role was not right. Chandrasekharan in fact backed Harish-Chandra till the very last stage of the committee's deliberations. At the last stage he yielded to the majority view.

I saw Harish-Chandra for the first time when he visited TIFR for a few days in late 1964. I was a junior member of the faculty and all of us on the faculty as well as the students were ushered into his presence. Some seniors engaged him in a conversation while the younger ones just watched. It was 2 years later that I met him for the first time. I was at the Institute for Advanced Study as a visitor and had gone to take my afternoon Tea where he too had come. I went up to him and introduced myself. He said 'Ah, yes, Borel told me that you have done some interesting work, but I do not understand this business about cohomology' or something to that effect. That made my day alright. After the first meeting I would meet him once in a while. We would talk about various things, in particular, Indian politics and the state of mathematics in India. He put me at ease, there was no trace of the kind of condescension that is a common experience in the Indian milieu. He expressed his strong views forthrightly. I never talked mathematics to him though I had embarked on reading his papers on representation theory. This was partly due to my diffidence when facing a giant intellect. More to the point, was a snub I had received from Andre Weil to whom I had foolishly said that I could improve some of Harish-Chandra's proofs: Weil said 'Local simplifications are always possible; they have little significance!'

On most issues my views were no different from Harish-Chandra's. However, I did not share his negative perceptions about Bhabha and Chandrasekharan. As Langlands put it, his ascetic temperament prevented him from seeing the positive qualities in the high-living Bhabha. He was even more scathing in his comments about Chandrasekharan than about Bhabha. In fact, he went to the extent of calling Chandrasekharan a liar. He had also encountered some pettiness in Chandrasekharan. During his 1964 visit to TIFR, he arranged for two of TIFR's junior faculty to visit the Institute for Advanced Study in 1965-66; and Chandrasekharan did not hide his resentment over it being done 'over his head'! Chandrasekharan had his faults, but there were many things about him that I admired. In a short span of a decade and a half, he had turned TIFR from the fledgling it was in 1949 (when he joined it) into a world renowned centre for mathematics. I could also see that in his powerful position he would have often been obliged to be evasive, laying him open to the charge of lying. Curiously, years later during a long chat I had with him (or rather a monologue from him) in Zurich, Chandrasekharan told me that he had always admired three people: Jawaharlal Nehru, Homi Bhabha and Harish-Chandra! I do not think he was lying - he had been in retirement for almost a decade.

Harish-Chandra invited me home for a meal - I do not remember if it was lunch or dinner. Nor do I remember who the other guests were. It is very likely that my Indian colleagues - 3 of them were in Princeton at that time – were present. I met Lily Harish-Chandra for the first time and had a taste of her gracious hospitality. She was in a sari and of course beautiful. Looking back, it surprises me that I have never seen her sporting any other dress except in the wedding photographs above. I did not meet her again though I would see her occasionally during the year. It is long after Harish-Chandra died that I got better acquainted with her and my wife became a good friend of hers.

During my stay at IAS, Harish-Chandra gave a long course on some work of Langlands. At one point, during the course of the lecture he said that Langlands uses some 'Ascoli's Theorem', to prove a result, but he can prove it in his own way and asked his audience if any one knew what this Ascoli's theorem was. The audience burst out laughing - Ascoli's theorem was standard fare in a graduate or advanced under-graduate course in American Universities. Harish-Chandra never went through such a course. He had essentially found a version of the Ascoli theorem himself.

Another amusing incident happened during his lectures, once again caused by his indifferent training before he embarked on mathematics. At one point he stated a lemma with two parts and embarked on its proof. After some ten minutes at it, he said 'So that proves the first part'. Borel who was in the audience burst out with 'My God, I thought you were proving the second part- the first part is trivial'. Harish-Chandra was visibly annoyed and said 'For some people some things are trivial; for others other things are trivial'. The first part of the lemma was algebraic in character while the second was analytic. Borel was much more at home with algebra than analysis.

Jaques Tits

Jaques Tits, a Belgian mathematician visited IAS for a few days and gave a talk on 'p-adic Lie Groups'. In his talk Tits was repeatedly mentioning 'compact unipotent' groups. At some point Harish-Chandra burst out with 'What do you mean! Compact unipotent groups are trivial'. Before the speaker could respond Andre Weil who was in the audience came up with 'Oh, Harish always thinks of the pathological case of real numbers!' causing a great deal of mirth. That needs some explanation: Mathematicians have invented a host of number systems – in fact one for each prime p - called p-adic numbers. They behave very differently from the familiar real number system; Tits was talking about Lie groups in a p-adic system. In the context of their being infinitely many p-adic systems, real numbers should be considered pathological!

After the year I spent in Princeton, I got to meet Harish-Chandra only in January 1973. It came about in the following way. The US food aid programme PL480 ended with a lot of unspent money in 1970 or there-abouts. The US agreed to a suggestion that the money may be spent in India on some joint programmes in Education. Binational conferences on Education in different fields were held. There was one on Mathematics in 1973 - the last in the series - that was held in July 73 in Bangalore. I was a member of the 'Steering Committee' for the conference. The committee met in early January 1973 to discuss issues connected with the organization of the conference. I was at the meeting. My senior colleague at TIFR, K. G. Ramanathan, who was also a member of the committee, for some reason, could not attend the meeting. Harish-Chandra was in Delhi around that time on a personal visit and the Chair of the committee, P. L. Bhatnagar, came to know of it. He requested Harish-Chandra to address the Steering Committee at the beginning of the meeting. As it turned out, Bhatnagar himself was unable to attend the meeting. Harish-Chandra came to the meeting. He began his address saying that he was going to be blunt, and blunt he was. His opening sentence was to the effect that no good work in mathematics was going on in India except at TIFR. He lambasted the poor quality of the mathematics in the universities and institutions other than TIFR pursuing research in mathematics. I was the lone person from TIFR at the meeting and felt very uncomfortable. After his diatribe against the universities, Harish-Chandra went on a scathing attack on TIFR. He criticized TIFR for keeping away from the universities instead of working with them and help them achieve higher standards. He was harsh on Chandrasekharan as the architect of the policy pursued vis a vis the universities. The criticism of TIFR, I was willing to admit had some merit, but Harish-Chandra had no idea of the ground realities of the relations between TIFR and the universities. In any case after that shelling of TIFR, I felt more comfortable about interacting with my fellow committee members. Harish-Chandra left after his brief speech and the committee started its discussions.

The Chairman Bhatnagar came to the next meeting of the Steering Committee. He began the meeting chiding his colleagues for not responding to Harish-Chandra's very uncharitable comments about 'one of our distinguished colleagues'. This was really amusing as everyone knew that there was no love lost between Chandrasekharan and Bhatnagar. The latter was very powerful and wielded considerable clout in the Indian University network as well as ministries in the government of India. TIFR however was out of his reach and Chandrasekharan wielded a lot of influence on the international scene. Each was resentful of the other over the clout the other wielded.

The 1973 visit to India was Harish-Chandra's last visit to the country. Shortly after his return to Princeton from that trip he suffered a heart attack. He was unwilling to make a trip to India after that as he felt that medical facilities in the country would not be able to meet his needs in the aftermath of that setback to his health.

In 1982 (September, I think), I received a letter from Harish-Chandra. I had never before corresponded with him, so I was surprised to receive a letter. The contents of the letter was an even greater surprise, and a very pleasant one at that. In the letter Harish-Chandra told me that he wants to nominate me for the Fellowship of the Royal Society. For that, he needed a letter from me stating that I will not turn down the Fellowship if elected. He also needed a few personal details about me. I was in the seventh heaven. I knew that Harish-Chandra had a good opinion of me as a mathematician, but this meant a much higher estimation than what I thought he had of me; and that a man of his temperament would take the trouble collecting supporters and filling forms which the nomination demanded was indeed flattering. I need not add that I responded promptly with all he wanted. Incidentally Harish-Chandra himself had been elected to the Royal Society in the early 70s. The physicist M. G. K. Menon once told me that he had nominated Harish-Chandra! After that I did not hear anything from Harish-Chandra. I met him, as I said shortly before his death in 1983. I was not sure if he eventually nominated me but I did not feel equal to asking him. Much later in 1984, I asked an Indian FRS if I had been nominated. He snapped back 'How do you mean - you don't know; you must have signed a letter saying that you wouldn't turn it down'. He would not give me time to explain but went on to tell me that I was nominated and reeled off a few more Indian names who had been nominated! In 1988 I got a letter from Robert Langlands asking for an up-date of my CV. As Harish-Chandra had passed away it was his responsibility as the seconder to update my CV for the Royal Society. I learnt that the two other who supported my candidacy were M. F. Atiyah and J. F. Adams. My nomination by Harish-Chandra was valid till 1990, but I did not get elected in that period. I was renominated by C. S. Seshadri in 1998 and got elected in the year 2000.

I was made the Chair of the Governing Council of the Mehta Institute of Mathematics and Mathematical Physics (now Harish-Chandra Research Institute) in 1987 (and continued as such for the next 30 years) located in Allahabad (now Prayagraj). It occurred to me in 1992 that as Allahabad is where Harish-Chandra cut his scientific teeth, it will be appropriate to install a bust of his in that institute to honour his memory. The idea was welcomed by my fellow members of the Council, in particular the Director H. S. Mani. With help from the late Professor S. M. Chitre, we located a sculptor, Mr Sonavadekar, to make the bust. Sonavadekar had made the statue of Vivekananda in Kanyakumari. We got in touch with Sonavadekar for making the bust. He was willing to undertake the job for a fee of Rs. 50,000/-. He went on to tell me that as he had never seen the man and had to work out of photographs, it would be difficult for him to capture the personality right; and so he would be happy only if his work was approved by someone who had known Harish-Chandra over a long period. He added that he would accept payment only after such an approval came! I was impressed by the man's integrity. Evidently there could be no one better than Lily to look at the bust to say whether it did justice to her husband's personality. I wrote to her and she promptly agreed to go to India to see the bust.

Lily came to Mumbai in early1993 when Sonawadekar had readied a clay model (from which the Bronze bust is to be cast). Lily and I went to see the clay model. Neither of us felt that he had captured Harish-Chandra adequately. Lily felt that the sculptor had 'Maharashtrianised' the face and I agreed with her: the model reminded me of a bust of the industrialist Kirloskar which I had seen. We had a long discussion which ended with Lily saying graciously that despite our reservation approval may be given; in a few decades from now there would be no one around who had actually seen Harish-Chandra; and for those around it would serve the cause of preserving his memory well enough even if it was not a good likeness.

On October 11, 1993 Harish-Chandra's bust was unveiled by C. S. Seshadri. Present on that occasion as honoured guests were family members of Harish-Chandra: Lily Harish-Chandra, Premala Chandra, Harish-Chandra's elder daughter, Dr. Kurien, his younger son-in-law, his brother Suresh Chandra and his wife. The then Chairman of Atomic Energy Commission, Dr. R. Chidambaram, presided over the occasion. Armand Borel and Mrs Gaby Borel were the other guests of honour. Seshadri, Chidambaram, Borel, Premala Chandra, myself and H. S. Mani (Director of MRI) spoke on the occasion. There was a tree plantation ceremony after that. The unveiling of the bust was a valedictory of sorts of a mathematics conference commemorating Harish-Chandra's 80th birth anniversary. With the exception of Armand Borel all speakers were from India. All the participants attended the ceremonial unveiling. The Proceedings of the conference have been published. Apart from the technical talks, the volume also carries the transcripts of the speeches made on the occasion of unveiling the bust.

Some 30 years after that bust was installed, Raghuram, a student of mine who was then heading the Math department at IISER Pune contacted me to ask me if IISER could get a copy of the bust. I told him about my reservations about the bust and suggested that he try to get one made by Charlotte Langlands. Charlotte I had discovered is a sculptor and she had known Harish-Chandra. I had seen a bust of A. Weil made by her for the Institute for Advanced Study. A bust of Harish-Chandra made by her is now installed at IISER. I had the privilege of unveiling the bust. I think Charlotte's bust captures Harish-Chandra somewhat better than Sonavadekar's.

In 2003, the MRI council thought of renaming the institute after Harish-Chandra. One reason for this was the talk going round about bringing in private funding of government funded scientific institutions. After an initial donation of 10 lakhs, the Mehta family (after whom the institute was named) had not made any financial grants to the institute. After 1968, the government of India was funding the institute fully. If private funding is to be got, the name Mehta Research Institute may be a disadvantage: one may be more successful if the institution was named after the great mathematician Harish-Chandra. But changing the name would need the consent of the Mehta family which had two members on the Governing Council. Professor Mani and I went to Kolkata to meet and talk to the Mehta family, the two members of the Governing Council in particular. The family was very gracious in giving their consent to the name change. The only question they asked was mildly embarrassing to me. They wanted to know if there was not a case for changing the name of the Tata Institute of Fundamental Research (to which I belonged). I admitted that there was indeed a case, but I was not in a position to initiate action on that.

In October 2023, the Harish-Chandra Research Institute organized a conference on Representation Theory to commemorate the birth centenary of Harish-Chandra. The invited speakers included eminent mathematicians R. Parthasarathy, Dragan Millicic, Laurent Clozel, Dipendra Prasad, and David Vogan. I was also one of the speakers.

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2. Transforming Teaching of School Mathematics - Making it Effective and Interesting

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The education in general, and the mathematics education in particular, has been a matter of great interest, serious concern, intense deliberations and endless debate. Many Committees and Commissions have been set up, a number of recommendations have been made and detailed reports have been prepared. The Kothari Commission - 1956, the Education Policy - 1986, and the National Education Policy - 2020, have been landmarks in this direction. This has resulted, over years, in revision of syllabi, writing and rewriting of textbooks and reforms in examination system. The efforts have certainly yielded some positive results. New language, new concepts and new topics have been introduced. I was introduced to coordinate geometry only in the 13th year of my education and did not know the meaning of the word 'set' when I passed my M.Sc. examination. Now every school child is familiar with coordinates and sets. Our students are doing well in Mathematical Olympiads, programmes for talented students are organised and NCERT brings out a range of good text books. But still there is a lurking feeling in the masses that everything is not well with the school mathematics education.

There may be several factors responsible for this situation: Shortage of teachers, lack of infrastructural facilities etc. These are all valid points, but we do not intend to discuss them here. We would like to discuss some changes that can be brought in class room teaching of mathematics to make it more effective without disturbing the basic structure of syllabus, text books and examinations. There are two main drawbacks hurting the education today. Failure to motivate and failure to generate interest. Students are not motivated to learn. They learn mathematics because it is there and learning it is necessary to pass the examination. Secondly, they find teaching of math uninteresting. The lack of interest results in lack of understanding, which, in turn, leads to cramming for passing in the examination. The purpose of education is defeated.

What can be done? The teachers can play an important role in correcting the situation. Our mathematics education, at present, is not only guided but even governed by the syllabus and the textbook. Since the syllabus is framed and the textbooks are written by experts, this method of teaching sounds perfect. Or is it perfect? Late Prot. A. M. Vaidya, professor and head of the department of Mathematics of Gujarat University, hinted at the imperfection of this method, by an illustration, in his presidential speech in Gujarat Mathematical Association's annual conference. He said: Imagine yourself, travelling by day time train from Ahmedabad to Mumbai. One way to travel is to close yourself in a compartment, dose off from time to time and complete the journey. Another way is to look out from the window from time to time, enjoy the view of hills, rivers and fields, see what is happening at the intermediate stations, converse with the fellow passengers. In either case, you will reach your destination, but in the first case, the journey will be uneventful and tiring; In the second, pleasant and relaxing. When a teacher teaches in a class room within a rigid framework of syllabus and textbooks, he takes the students on a journey of learning in the first way. The destination of completing the syllabus is reached, but the journey is boring, may be even painful. Then what could be another way of teaching, where the students would find the journey less stressful and relaxing. This is what we would like to discuss in what follows.

2.1 Looking out of a window

The other way is to look out of a window. There is so much material in mathematics and that related to mathematics, which may not be a part of any syllabus, but quite relevant to teaching. This material can be presented to the students along with the usual textbook material in an unobtrusive way. The presentation of such material will lessen the dryness of teaching and will motivate the students to study mathematics. The idea is not new. In every article about improvement of mathematics education, we read about motivating students by giving applications and by relating teaching with day-to-day experiences. We simply elaborate on this theme here. One may argue that teachers are already trained for classroom teaching during their B.Ed. study. This is true, but the flavour there is different. There the students learn about philosophy of teaching and learning, psychology of students, planning of lessons based on a textbook, techniques of evaluation etc. Our suggestion is to supplement the teaching by material and techniques, which would enhance the interest of students. Needless to add, such material should be served in appropriate doses, at appropriate places, so that the mainstream teaching is not disturbed.

2.2 Out of syllabus, outside the text book

The following topics may serve as a basis for the material meant to help the enrichment programme of the class room.

1. History	2. Anecdotes and stories
3. Interesting results	4. Patterns and beauty
5. Games and puzzles	6. Applications
7. Mathematics in the world around us	8. Activities
9. Models and charts	10. Use of technology

We briefly elaborate on each of the above topics.

2.2.1 History

The history of mathematics should not be taught as a separate topic at school level, but some glimpses of historical nature of development of mathematics can be given while teaching relevant topics. It may be told that arithmetic and geometry were developed in very early times by the Babylonians, Egyptians and Indians for the use in commerce, agriculture and construction of structures. Indian contribution to mathematics can be duly emphasized and it can be explained how very useful and fundamental concepts of numbers reached Europe, via Arabs, from India. While teaching geometry, it should be brought to the notice of the students how Euclid built a beautiful edifice of geometry with the help of logic. The life stories of mathematicians whose work is related to school mathematics, such as Pythagoras, Euclid, Brahmagupta, Bhaskaracharya II and others can be narrated briefly. It should be mentioned that mathematics is a humanistic activity, carried out in response to human needs, human curiosity and search for beauty and harmony. The universal nature of mathematics and the story of its development cutting across geographical boundaries and spanned over four millennia will help the students to get the right perspective of the subject.

2.2.2 Anecdotes and stories

There are a number of stories associated with mathematicians and mathematical incidents, not always true, but interesting and instructive all the same, which the students would relish to listen. Since the stories we mention below are well known, we do not give details.

(1) Euclid, when king Ptolemy asked for a short cut to learning geometry, said ‘There is no royal road to geometry’.

(2) Euclid, when a businessman asked what he would gain by learning geometry, asked his servant to give a kopek to the business man because he likes to gain from everything that he learns.

(3) Archimedes ran in the streets of Athens, naked, shouting ‘Eureka, Eureka’.

(4) Archimedes, when a soldier’s shadow fell on his geometrical figures, said, ‘Do not disturb my diagrams’ and was beheaded by the enraged soldier.

(5) Pythagoras passing by a den of a blacksmith and discovering the relation between mathematics and music

(6) Lilavati missing the right moment of marriage

(7) Hamilton's discovery of quaternions while walking on a bridge
(8) Gauss, summing up the numbers from 1 to 100, quickly at the age of seven
(9) Fermat's Last Theorem
(10) Ramanujan number 1729

It may be remembered that story telling, while an effective tool if used with discretion, could prove counterproductive if overused.

2.2.3 Interesting results

There are many interesting results and ideas in mathematics, which, while not part of syllabus, can be understood and appreciated by high school students. Mentioning such results will arouse the student's curiosity to know more. For example, while teaching the construction of a bisector of an angle, it may be pointed out that an angle cannot be trisected in general by an unmarked ruler and a compass. While teaching regular polygons, it may be mentioned that only five regular polyhedra exist. Even the models of five regular polyhedra may be shown. While teaching the theorem of Pythagoras, talking about Pythagorean triplets, or even Fermat's Last Theorem, may be a fruitful diversion. Here are a few more examples:

1. Königsberg seven bridges problem
2. Cycloid as a curve of quickest descent
3. The set of prime numbers is infinite
4. Fibonacci sequence
5. Golden ratio
6. Perfect and amicable numbers
7. Magic squares
8. Russel's paradox

2.2.4 Patterns and beauty

Patterns play a very important role in mathematics and students should be made aware of this fact from school days. The patterns are pleasing to the mind, and recognition of a pattern can lead to a general result. Let us see some examples of patterns which the school students can enjoy and appreciate.

(1) The table of 9 has a pattern.
(2) The binomial coefficients form a pattern.
(3) The last two digits of squares of numbers form a pattern. For example, the last two digits of $1^2, 11^2, \dots, 91^2$ are $01, 21, 41, 61, 81, 01, \dots 81$ respectively. The last two digits of cubes of numbers also form a pattern. For example, the last two digits of $02^3, 12^3, 22^3, \dots, 92^3$ are $08, 28, 48, 68, 88, 08, 28, 48, 68, 88$ respectivly.
(4) We have $1 + 2 + 3 + \cdots + n = \frac{1}{2}n(n + 1)$;
$1.2 + 2.3 + \cdots + n(n + 1) = \frac{1}{3}n(n + 1)(n + 2)$;
$1.2.3 + 2.3.4 + \cdots + n(n + 1)(n + 2) = \frac{1}{4}n(n + 1)(n + 2)(n + 3)$.
(5) It would be interesting to find and prove the general form of the following equalities of sums of numbers:

$$1 + 2 = 3;\ \ 4 + 5 + 6 = 7 + 8;\ \ 9 + 10 + 11 + 12 = 13 + 14 + 15.$$

Students may be told that while finding a pattern is interesting, sometimes we come across a situation where there is no pattern even though we may expect one. One such example is the sequence of prime numbers $2, 3, 5, 7, 11, 13, 17, 19, \dots$, which behaves very erratically.

Patterns are pleasing and beautiful, but the beauty crops up in mathematics in other ways too. The beauty may lie in simplicity of proof, or in elegance of a result, or in a result which is surprising.

The proof that $\sqrt{2}$ is irrational is beautiful, and so is the proof of Euclid to show that the set of primes is infinite.

The result $e^{i\pi} + 1 = 0$ puts all important numbers $0, 1, e, \pi$ and i together in one equation. What a beauty!

And think of Fermat's result: Every prime of the form $4n + 1$ is a sum of two squares!

If this is not surprising, what is?

Just as a tour guide shows to the tourist everything, that is worth seeing, explaining its hidden significance, so the teacher should draw the attention of a student to the charm and the beauty of the results he teaches.

Shri Rabindranath Tagore, said that the teacher should not merely inform, he should inspire.

We can say that a mathematics teacher is not there to teach dryly, his task is to bring life to dry facts by showing their intrinsic beauty.

2.2.5 Games and Puzzles

There is plenty of literature available on this topic, in books, magazines and even newspapers. One reason is that the mathematical games and puzzles are considered to be a source of entertainment and it is this aspect that makes them popular. This is fine and this aspect should be exploited to make the teaching interesting. But the educational aspect of games and puzzles should not be lost sight of. The teacher should make the students play a game and find a winning strategy. The students should be told that finding one solution of a puzzle is not enough. They should analyse a problem and think of a path or paths leading to a solution. Also, it is believed that the puzzles and problems are for talented students only. The teacher should give challenging problems to the talented, but should coin diluted ones for average students. Being able to solve a problem increases a student's confidence and he feels encouraged to learn more. It is the teacher's task to build up and increase the confidence of every student.

2.2.6 Applications

One of the reasons why a student feels disenchanted with mathematics is that he does not know what it is used for, if it has any use at all. So, giving applications of mathematics in the classroom will motivate the students to learn the subject. We can tell the students that the nature uses mathematics to get the best results and man uses mathematics to understand nature on one hand and to increase his efficiency and comforts on the other.

The velocities and forces combine as per the parallelogram law, the path of motion of a particle is linear, circular or parabolic according to the initial conditions. The shape of the Earth is oblate spheroid and its orbit round the Sun is elliptical. When a ray of light falls on a plain mirror, it is reflected in such a way that the angle of incidence equals the angle of reflection and this can be explained by simple geometry. It is this principle of reflection of light, combined with a property of a parabola, that is used to make the shapes of light reflectors and dish antennas paraboloidal. The phenomenon of a whispering gallery can also be explained by the reflection of sound waves from an elliptic surface. The spherical shape of a raindrop, the catenary shape of a cloth line and the cycloidal shape of the curve of quickest descent can all be explained by mathematics.

Way back in the third century B. C., Eratosthenes calculated the circumference of the Earth by using a simple geometrical result. In the nineteenth century the British astronomer John C. Adams and the French astronomer Le Varreer discovered the planet Neptune only by mathematical calculations on paper. What greater power and usefulness of mathematics can be there! The calculations of Eratosthenes for finding the circumference of the Earth are quite simple and can be presented in a class room. The story of discovery of Neptune is quite exciting and must be narrated to the students.

2.2.7 Mathematics in the world around us

Mathematics in the world around us: The school mathematics deals with numbers and shapes. Both of these are found in abundance in the world around. A student can be asked to collect the grocery, electricity and restaurant bills of his house and check their calculations. He can look at income tax return and bank statement of his father (if he is willing to show them). He may as well write a cheque or fill in a deposit slip. He will find that all the medical reports of his grandparents regarding blood pressure, diabetes and haemoglobin level are full of numbers. He will find numbers all over his TV screen, while watching an ODI cricket match. It may be of interest for him to know that Duckworth and Lewis, whose names have been associated with the D-L system, used to decide the result of an international cricket match when the play is curtailed, were, respectively, a statistician and a mathematician. He may have noticed that a clock, a calendar, a thermometer and a TV remote control display numbers and cars, trains, aeroplane flights are identified by numbers. Also, Aadhaar card, Pan card, pass port and bank account all carry numbers. He must be reading about five trillion dollars economy, seven percent growth, 1.4 billion people. A casual look at a newspaper will convince the student about dominance of numbers in daily life. Whether it is a page on sports or on business, whether the news is about elections or environment, it will be the numbers and graphs that will tell the story.

Also, a student will find that books, balls, dices, water pipes, tree trunks, constellations in the sky, bicycle wheels, cells of beehives all have geometrical shapes and he will find geometrical designs at unexpected places-hospitals, banks and airports. He may find that the shapes of clouds, mountains and seashores do not match with any regular geometrical shapes he is studying in school. These shapes come under the category of what are called 'fractals'. The Euclidean geometry is useful in studying the geometry of fractals.

2.2.8 Activities, models and charts

'I hear and I forget, I see and I remember, I do and I understand', goes an adage. So it is helpful if the class room lectures are supported by visuals and activities. Students can use counters or beads to understand addition and multiplication of numbers. They can make geometrical figures by using straws or bicycle spokes or by cutting cardboards or by folding paper. This will help them in identifying geometrical figures as well as in understanding their properties. By drawing a figure on a graph paper and calculating its area by counting squares, a student can verify the formula for area.

The students can make paper models of a cylinder, a cone, a torus and a Möbius band and study their properties. They can be shown the models of regular polyhedra, prisms and pyramids and may be guided to make such models. Students can make charts of mathematicians and of history of development of mathematics. They can make posters of interesting patterns of numbers and of mathematical puzzles. Also, the posters displaying results of algebra, geometry and trigonometry be prepared. A room may be designated as a 'Mathematical Activity Room' in school, which is equipped with marbles, dice, play cards, straws, bicycle spokes, card boards, paper pieces and scissors. The charts and posters should be displayed on walls. Some readymade models may help, but are not necessary. The students should be able to play around in this room, experiment, solve puzzles, play games, discuss, study charts and posters, make their own models and talk freely with their teachers.

2.2.9 Use of Technology

Many short videos on topics related to school mathematics are now available. Such videos can be shown in the class room. The teacher himself can make a video and use techniques of GeoGebra or other such software and power point presentation to make his teaching interesting and effective and to save time.

2.3 Implementation

The implementation of the programme suggested above, though not difficult, will certainly require extra efforts on the part of teachers. There is no single book which would contain all the material needed. So it will be necessary to look into various sources. The good thing is that many books are available on history of mathematics, recreational mathematics and general topics of mathematics. (For example, [3], [5], [6], [7].) Also there is no dearth of books on problems and puzzles. ([2] is a good collection of puzzles, where hints and solutions are also provided. Also [1] contains some good puzzles.)

Vikram A. Sarabhai Community Science Centre, Ahmedabad has prepared kits of models and teaching aids for primary and secondary school students. [1] is a manual for models and teaching aids, written by late Prof. A. R. Rao, who was the pioneer of the concepts of a mathematical laboratory and nonformal mathematics. In [4], there are more than forty video demonstrations of mathematical models, with instructions for making them.

These days, many schools have facilities of smart boards, computers and internet. Then there are omnipresent mobile phones. Such devices can help in showing charts, graphs, tables and diagrams, which would be difficult to show otherwise. However, the overuse of technology should be avoided. Sometimes, display of a prepared lecture on the screen does not give enough time to students to understand, let apart to note down the main points. That way, the blackboard method is still the best for teaching at a deliberate pace.

One question that is often raised is: ‘Where is the time for such a programme?’ The answer is in understanding the objective of the programme. Adding new material is not the idea; it is only to give a new flavour to the existing method of teaching to create interest among students and to ward off their boredom. Historical events and stories can be told in brief, encouraging students to find details later. Some applications may be mentioned during the course of routine teaching and puzzles and games may be a part of mathematical club activity. The teacher, while preparing his lecture, should also think of material of nonformal mathematics that he would like to include in his lecture to make it effective and also plan as to when and how he would present it. Advance planning will help in focusing on delivering only the necessary details during the lecture.

The use of technology, too, will help in saving some time. Further, some states, like Gujarat, have recently reduced the syllabus of school mathematics. The teaching time thus saved may be used to peep out of the window and to talk about topics outside of textbooks.

Finally, training programmes for teachers may be arranged to discuss ways and means to make teaching interesting and learning enjoyable.

References

1. A. R. Rao, A Manual of Mathematical Models and Teaching Aids (Second Edition), Vikram A. Sarabhai Community Science Center, Ahmedabad, 2010.
2. A. R. Rao, Brain Sharpeners (Fourth Edition), Vikram A. Sarabhai Community Science Centre, Ahmedabad, 2011.
3. E. T. Bell, Men of Mathematics, Simon & Schuster, New York, 1937.
4. Hema Vasavada, Activities and Demonstration of Math models, www.youtube.com/@ Activities and demonstration 4195
5. Peter Tannenbaum / Robert Arnold, Excursions in Modern Mathematics, Prentice Hall, New Jersey, 1992.
6. Richard Courant and Herbert Robbins, What is Mathematics? (Second Edition), Oxford University Press, New York, 1996.
7. Simon Singh, Fermat’s Enigma, Anchor Books, New York 1997.

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3. What is Happening in the Mathematical World?

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3.1 The Best Versions of Iconic Shapes have been Identified

As far back as the 1930s, attempts were made to find the shortest possible rectangle that can be twisted into a Möbius strip. It seems intuitively clear that it is easy to twist a long, thin rectangle into a one-sided strip, but that doing so with a square is impossible. But where exactly is the boundary? In 1977, Basil Halpern and Jacqueline Weaver conjectured that to be twisted in a Möbius strip, a rectangle should have aspect ratio (the ratio of the length of a strip to its width) greater than $\sqrt{3}$.

Recently, *Richard Schwartz* of Brown University, Providence, Rhode Island, USA posted three results about optimal shapes in quick succession, starting last August, including one with his wife, *Brienne Elisabeth Brown*. All of these results are about minimizing the amount of paper, rope or string used to make a particular shape. Schwartz's recent run started with the Möbius strip, which is formed by taking a strip of paper, giving it a twist, and joining the ends.

In August 2023, Schwartz proved that Halpern and Weaver were correct: There is no way to twist a rectangle having aspect ratio less than $\sqrt{3}$, into a Möbius strip.

One central tool in figuring out what optimal shapes look like is called a "limiting shape". Limiting shapes are different in crucial respects from the shapes being optimized but also share some of their properties. In this case, a limiting shape for the Möbius strip is created as follows. Start with a rectangular flat piece of paper that is one unit wide and $\sqrt{3}$ units long. Begin by folding it following the instructions as indicated in the Figure 1. The first step involves creating a diamond. Then fold up across the midline of the diamond and tape together the two edges, shown by blue and yellow dotted lines, that meet in the interior of the diamond. Now adjust the tiniest bit of slack by making the strip just a little bit longer, or a little bit narrower, so that you can tug the triangles apart. This is your Möbius strip.

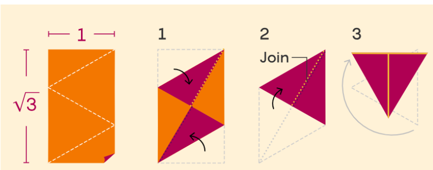


Figure 1

Schwartz combined arguments from topology and geometry. He used topology to show that on every paper Möbius strip it is possible to draw intersecting lines that form a T in a particular way. Then he showed that when such a T exists, the aspect ratio of the strip has to be greater than $\sqrt{3}$.

It has been known for long that such a triangle is a limiting shape for Möbius strips. However, in the present paper Schwartz showed that other limiting shapes that would allow for a stubbier strip do not exist. After his paper on the Möbius strip, Schwartz proved that the limiting shape of the twisted cylinder can be made by folding a 1-by-2 rectangle formed from four stacked right isosceles triangles as shown in the Figure 2. Schwartz then turned his attention to the three-twist Möbius strip. Like

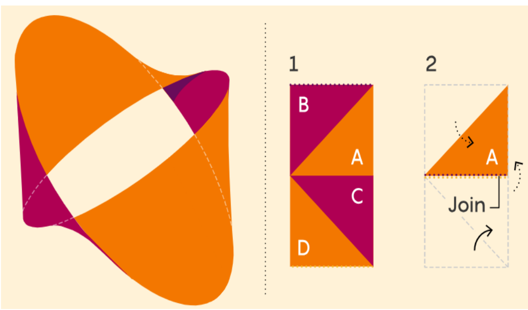


Figure 2

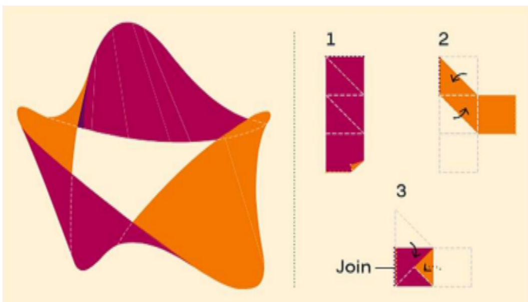


Figure 3

the one-twist strip, this is a one-sided figure, but because of the two extra twists, its boundary is more complicated. Schwartz's wife, *Brienne Elisabeth Brown*, found a construction she calls the "crisscross" (shown in Figure 3) that is a limiting shape of a three-twist Möbius strip which is three times as long as it is wide.

Brown and Schwartz also found a completely different limiting shape for the three-twist cylinder, which they call the cup, which cannot be made to lie flat. Brown and Schwartz explain why they think that the optimal three-twist strip has an aspect ratio of 3. But they have not yet been able to prove it, in part because the existence of the cup, which cannot be flattened, means that the kinds of arguments Schwartz made in the one- and two-twist cases cannot be extended to the three-twist case.

Source: *https://www.quantamagazine.org/mathematicians-identify-the-best-versions-of- iconic -shapes- 20240105/?mc_cid=169f0a8ce7&mc_eid=df6d259cb7*

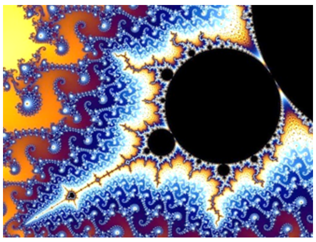
3.2 Celebrating 100th Birth Anniversary of Beno ît B. Mandelbrot, Creator of Fractal Geometry

This year is being celebrated as the birth centanery year of *Benoit B. Mandelbrot*, who is recognized for his contribution to the field of fractal geometry, which included creating the word "fractal", as well as developing a theory of "roughness and self-similarity" in nature.

Mandelbrot was born in Poland on 20 November, 1924. He was a Polish-born French-American mathematician having broad interests in the practical sciences, especially regarding what he labeled as "the art of roughness" of physical phenomena and "the uncontrolled element in life".

Mandelbrot studied mathematics, graduating from universities in Paris and in the United States and receiving a master's degree in aeronautics from the California Institute of Technology. He spent most of his career in both the United States and France. In 1958, he began a 35-year career at IBM, where he became an IBM Fellow. Because of his access to IBM's computers, Mandelbrot was one of the first to use computer graphics to create and display fractal geometric images, leading to his discovery of the Mandelbrot set, a two-dimensional set of complex numbers that exhibits an incredibly intricate and beautiful pattern, in 1980. He showed how visual complexity can be created from simple rules.

For example, a Mandelbrot set (say, M) shown in the following figure can be constructed using the following simple rule: Identify an area (a rectangle) in a complex plane. Discretize this area by selecting a grid of pixels of desired resolution. Each grid point, say (x, y) represents a complex number $c = x + iy$. To decide whether c is in the mandelbrot set or not, we can perform the following test. Generate a sequence $\{z_n\}$ of complex numbers taking $z_0 = 0$ and using the iterative formula $z_{k+1} = z_k^2 + c$. If this sequence diverges to infinity then c does not belong to M, otherwise it belongs to M. This test can be realized computationally by checking if $|z_k| > 2$ within 255 iterations.



Mandelbrot said that things typically considered to be "rough", such as clouds or shorelines, actually had a "degree of order". His maths- and geometry-centered research included contributions to such fields as statistical physics, meteorology, hydrology, geomorphology, anatomy, taxonomy, neurology, linguistics, information technology, computer graphics, economics, geology, medicine, physical cosmology, engineering, chaos theory, econophysics, metallurgy, and the social sciences.

Mandelbrot saw financial markets as an example of "wild randomness", characterized by concentration and long-range dependence. He developed several original approaches for modelling financial fluctuations.

As a visiting professor at Harvard University, Mandelbrot began to study mathematical objects called Julia sets (closely related to the Mandelbrot set and named after the French mathematician

Gaston Julia) that were invariant under certain transformations of the complex plane. Building on previous work by Gaston Julia and Pierre Fatou, Mandelbrot used a computer to plot images of the Julia sets. While investigating the topology of these Julia sets, he studied the Mandelbrot set which was introduced by him in 1979. The Mandelbrot set is indeed one of the most astonishing discoveries in the entire history of mathematics.

In 1975, Mandelbrot coined the term *fractal*. Fractals are infinitely repeating mathematical shapes; no matter how much you zoom in on them, the same irregular shape will repeat over and over at infinitely smaller scales.

Mandelbrot expanded and updated his ideas in his book entitled *The Fractal Geometry of Nature*, published in 1982 by W. H. Freeman and Company. This influential work brought fractals into the mainstream of professional and popular mathematics. Mandelbrot emphasized the use of fractals as realistic and useful models for describing many “rough” phenomena in the real world. He concluded that “real roughness is often fractal and can be measured.”

Mandelbrot also put his ideas to work in cosmology. He offered in 1974 a new explanation of Olbers’ paradox, relating to the problem of why the sky is dark at night. If the universe is endless and uniformly populated with luminous stars, then every line of sight must eventually terminate at the surface of a star. Hence, contrary to observation, this argument implies that the night sky should everywhere be bright, with no dark spaces between the stars. This paradox was discussed in 1823 by the German astronomer Heinrich Wilhelm Olbers. Mandelbrot demonstrated the consequences of fractal theory as a sufficient, but not necessary, resolution of the paradox. He postulated that if the stars in the universe were fractally distributed, it would not be necessary to rely on the Big Bang theory to explain the paradox. His model would not rule out a Big Bang, but would allow for a dark sky even if the Big Bang had not occurred.

Mandelbrot’s awards include the Wolf Prize for Physics in 1993, the Lewis Fry Richardson Prize of the European Geophysical Society in 2000 and the Japan Prize in 2003. The small asteroid *27500 Mandelbrot* was named in his honor. He received more than 15 honorary doctorates and served on the board of many scientific journals.

Mandelbrot died at the age of 85 in Cambridge, Massachusetts, on 14 Oct. 2010.

Sources:

1. *https://en.wikipedia.org/wiki/Benoit_Mandelbrot*
2. *https://aurora-israel.co.il/en/Benoit-Mandelbrot-mathematician-known-for-his - work -on - fractals/*

3.3 New Mathematical Approach helps to Solve Stochastic Partial Differential Equations

Whether it is physical phenomena, share price fluctuations or climate changes - many dynamic processes in our world can be described mathematically with the help of partial differential equations (PDEs). Such PDE based models are not adequate to explain the randomness in the dynamic process under consideration. When randomness in these processes is taken into account, the models are represented in terms of stochastic partial differential equations, something researchers have been working on for some decades now.

Stochastic partial differential equations can be used to model a wide range of dynamic processes, for example, the surface growth of bacteria, the evolution of thin liquid films, or interacting particle models in magnetism. However, these concrete areas of application play no role in basic research in mathematics as it is always the same class of equations which is involved.

Mathematicians are concentrating on solving the equations in spite of the stochastic terms and the resulting challenges such as overlapping frequencies which lead to resonances.

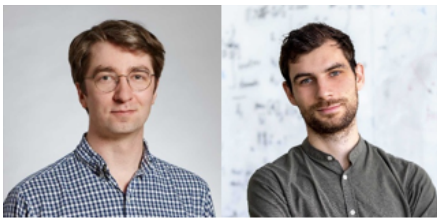
Working together with other researchers, *Dr. Markus Tempelmayr* of the University of Münster has found a new method which helps to solve a certain class of such equations.

The basis for their work is a theory by Prof. Martin Hairer, recipient of the Fields Medal, developed in 2014 with international colleagues. They applied tools from the fields of stochastic analysis, algebra and combinatorics. This theory is seen as a great breakthrough in the research field of singular stochastic partial differential equations (SSPDEs). The general form of SSPDE is as follows:
$du(t,x) = L(u)(t,x)dt + F(t,x,u(\mathbf{t},\mathbf{x}))dW(t)$, where $u(t,x)$ represents the unknown stochastic process defined on a time-space domain $[0,T] \times D, \mathbf{L}(\mathbf{u})$ is a nonlinear operator (e.g., Laplacian, fractional Laplacian) that may involve singular coefficients. $\mathbf{F}(\mathbf{t},\mathbf{x},\mathbf{u})$ is a nonlinear mapping that may exhibit singularities and $\mathbf{W}(\mathbf{t})$ is a Wiener process or other stochastic process representing noise.
Applications of SSPDEs include, modeling phenomena with localized sources or discontinuities, such as turbulence, quantum field theory in Physics, describing financial models with jumps or sudden changes, Modeling biological processes with localized inputs or disturbances etc.
The theory developed by Prof. Martin Hairer has provided a complete 'toolbox,' so to speak, on how such equations can be tackled." The problem is that the theory is relatively complex, with the result that applying the 'toolbox' and adapting it to other situations is sometimes difficult.
Dr. Tempelmayr and researchers looked at aspects of the 'toolbox' from a different perspective and found and introduced a method which can be used more easily and flexibly. They took an analytical approach, and in particular, worked on the question of how the solution of the equation changes if the underlying stochastic process is changed slightly. The approach they took was not to tackle the solution of complicated stochastic partial differential equations directly, but, instead, to solve many different simpler equations and prove certain statements about them. The solutions of the simple equations can then be combined to arrive at a solution for the original complicated equation. The results have been published in the journal *Inventiones mathematicae.*
The study, in which Tempelmayr was involved as a doctoral student under Prof. Felix Otto at the Max Planck Institute for Mathematics in the Sciences, was published in 2021 as a pre-print. Since then, several research groups have successfully applied this alternative approach in their research work.

Sources:

1. *https://phys.org/news/2024-06-mathematical-proof-equations-random-components.html*
2. *https://link.springer.com/article/10.1007/s00222-024-01275-z*

3.4 The Fastest Possible Flow Algorithm has been Developed



In a breakthrough, the ETH Zurich researchers *Rasmus Kyng* and *Maximilian Probst Gutenberg* and their team have developed a superfast network flow algorithm, which tackles the question of how to achieve the maximum flow in a network while simultaneously minimising transport costs. Their algorithm can be applied to calculate the optimal, lowest-cost traffic flow for any kind of network - be it rail, road, water or the internet. The algorithm performs these computations so fast that it can deliver the solution at the very moment a computer reads the data that describes the network!
Even though researchers have been working on this problem for some 90 years, previously, it took significantly longer to compute the optimal flow than to process the network data. And as the network became larger and more complex, the required computing time increased much faster, comparatively speaking, than the actual size of the computing problem.
Kyng and Gutenberg's approach eliminates this problem. Using this algorithm, computing time and network size increase at the same rate. A glance at the raw figures shows just how far the improvement has been achieved: until the turn of the millennium, no algorithm managed to

compute faster than $m^{1.5}$, where m stands for the number of connections in a network that the computer has to calculate (network size), and just reading the network data once takes m time. In 2004, the computing speed required to solve the problem was successfully reduced to $m^{1.33}$. Using this new algorithm, the additional computing time required to reach the solution after reading the network data is now negligible.

Two years ago, Kyng and his team presented mathematical proof of their concept in a groundbreaking paper, which won the 2022 Best Paper Award at the IEEE Annual Symposium on Foundations of Computer Science (FOCS). The first algorithm was focused on fixed, static networks whose connections are directed, meaning they function like one-way streets in urban road networks. The algorithms published this year are now also able to compute optimal flows for networks that incrementally change as new connections are added over time. Furthermore, in a second paper accepted by the IEEE Symposium on Foundations of Computer Science (FOCS) in October, the ETH researchers have developed another algorithmthat also handles connections being removed. Specifically, these algorithms identify the shortest routes in networks where connections are added or deleted, as it often happens in the real-world traffic networks.

The team also uses and designs new mathematical tools that speed up their algorithms. In particular, they have developed a new data structure for organizing network data; this makes it possible to identify any change to a network connection extremely quickly; this, in turn, helps make the algorithmic solution so amazingly fast.

Sources:

1. *https://www.eurekalert.org/news-releases/1049865*
2. *https://dl.acm.org/doi/10.1145/3618260.3649745*

3.5 Geometric Langlands Conjecture has been Proved

A group of nine mathematicians have proved the geometric Langlands conjecture, a profound mathematical statement that establishes a deep connection between two seemingly disparate fields: number theory and algebraic geometry.

It is a cornerstone of the broader Langlands program, originated by *Robert Langlands* in the 1960s, a vast network of interrelated conjectures that aim to unify various branches of mathematics. The Langlands program holds in three separate areas of mathematics: number theory, geometry and what are called function fields.

Some of the key interrelated conjectures within the Langlands program, include: Classical Langlands Correspondence, Local Langlands Correspondence, Global Langlands Correspondence, Hodge Conjecture, Tate Conjecture etc.

The geometric Langlands conjecture essentially states that there exists a correspondence between: Galois representations over a function field and Automorphic forms on a certain group associated with the function field.

The geometric Langlands conjecture was proved for $GL(1)$ by Pierre Deligne and for $GL(2)$ by Drinfeld in 1983. Laurent Lafforgue proved the geometric Langlands conjecture for $GL(n, K)$ over a function field K in 2002.

A claimed proof of the categorical unramified geometric Langlands conjecture was announced on May 6, 2024 by a team of mathematicians led by *Dennis Gaitsgory* (left) and *Sam Raskin* (right) of Yale University. The proof occupies more than 1,000 pages across five papers and has been called "so complex that almost no one can explain it".

Gaitsgory has dedicated the past 30 years to proving the geometric Langlands conjecture. Over the decades, he and his collaborators have developed a massive body of work on which the new proof rests.

In the years leading up to the proof, the research team created not one but many routes into the heart of the problem. The understanding that they have developed is so rich and so broad, that they have encircled the problem from every direction. It had no way to escape.
Source: *https://www.quantamagazine.org/monumental-proof-settles-geometric-langlands-conjecture- 20240719/?mc_cid=6520b06369&mc_eid=df6d259cb7*

3.6 Estimate about Number of Zeroes of Zeta Function with Real Part at least 3/4, Improved

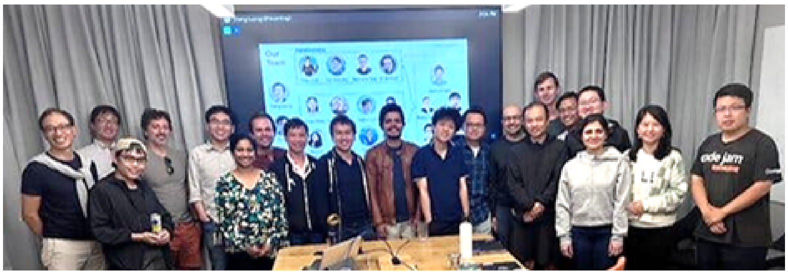
The Riemann hypothesis, is one of the famous open problems in Mathematics, whose solution comes with a $1 million reward from the Clay Mathematics Institute. Its proof would give mathematicians much deeper understanding of how prime numbers are distributed, while also implying a mass of other consequences.
The Riemann hypothesis is a statement about the Riemann Zeta function, ζ from $\mathbf{C}$ to $\mathbf{C}$, which is defined by $\zeta(s) = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots$, for $Re(s) > 1$, and its analytic continuation elsewhere. Leonhard Euler first introduced and studied the function over the reals in the first half of the eighteenth century. Bernhard Riemann's 1859 article "On the Number of Primes Less Than a Given Magnitude" extended Euler's definition to a complex variable, proved its meromorphic continuation and functional equation, and established a relation between its zeros and the distribution of prime numbers. It is relatively easy to show that all negative even numbers: $-2, -4, -6$ and so on, are zeroes of the Zeta function, so these are called trivial zeros. Riemann conjectured that 'all the other zeros of the function, called nontrivial zeros, have a real part of 1/2. This is the Riemann hypothesis, and proving it has been extremely difficult. But mathematicians can still get useful results, just by showing that possible exceptions are limited. In many cases, we can get new results about prime numbers from this.
It is relatively easy to show that every nontrivial zero must have a real part between 0 and 1. In 1940, an English mathematician named Albert Ingham was able to show that between $0.75 \leq x \leq 1$ there are at most $y^{3/5+c}$ zeros $(s = x + iy)$ with an imaginary part of at most y, where c is a constant between 0 and 9. In the following 80 years, this estimation barely improved. This estimate of Ingham, made it possible to apply the prime number theorem to short intervals of the type $[x, x + x^\theta]$, for $\theta > 1/6$.

In a breakthrough result posted online in May, *James Maynard*, Field Medalist-2022 and a number theorist from Oxford University (left) and *Larry Guth*, an expert harmonic analyst of the Massachusetts Institute of Technology (right) have succeeded in significantly improving Ingham's estimate for the first time. According to their work, the zeta function in the range $0.75 \leq x \leq 1$ has at most $y^{(13/25)+c}$ zeros with an imaginary part of at most y. The authors show in a quantitative sense that zeros of the Riemann zeta function become rarer the further away they are from the critical straight line. This makes it possible to reduce the size of the intervals for which the prime number theorem applies, to $[x, x + x^{2/15}]$ instead of $x, x + x^{1/6}$.
The new proof automatically leads to better approximations of how many primes exist in short intervals on the number line, and stands to offer many other insights into how primes behave.
Sources:

1. *https://www.quantamagazine.org/sensational-proof-delivers-new-insights-into-prime-numbers-20240715/*
2. *https://www.scientificamerican.com/article/the-riemann-hypothesis-the- biggest- problem-in-mathematics-is-a-step-closer/*
3. *https://en.wikipedia.org/wiki/Riemann_zeta_function*

3.7 Google Deepmind's New AI Systems can now Solve Complex Maths Problems

Google DeepMind team is working hard to generate AI systems for various tasks and subsequently improve their capabilities, since decades. The team has evolved AI systems like AlphaGo in 2016, for playing games like Go and Chess; AlphaZero in 2017, a more generalized version of AlphaGo; AlphaFold in 2018 to predict the 3D structure of proteins from their amino acid sequences; AlphaTensor in 2022 which was capable of finding faster Matrix multiplication algorithms; Alpha Geometry in January 2024 for solving problems in Geometry.



Now, Google DeepMind says it has trained two specialized AI systems to solve complex mathematics problems involving advanced reasoning. The systems - called *AlphaProof* and *AlphaGeometry 2* - worked together to successfully solve four out of six problems from this year's International Mathematical Olympiad (IMO), a prestigious competition for high school students. They won the equivalent of a silver medal. The picture shows Google DeepMind's "superhuman reasoning team". *Pushmeet Kohli*, vice president of research at Google DeepMind worked on this project.

There are a few reasons for maths problems that involve advanced reasoning to be difficult for AI systems to solve. These types of problems often require forming and drawing on ideas. They also involve complex ordered planning, as well as setting subgoals, backtracking, and trying new paths. All these are challenging for AI.

Bridging this gap was Google DeepMind's goal in creating AlphaProof, a reinforcement-learning-based system that trains itself to prove mathematical statements in the formal programming language Lean. The key is a version of DeepMind's Gemini AI that is fine-tuned to automatically translate maths problems phrased in natural, informal language into formal statements, which are easier for the AI to process. Automating the process of translating data into formal language is a big step forward for the maths community. This created a large library of formal maths problems with varying degrees of difficulty.

The Gemini model works alongside AlphaZero, to prove or disprove millions of mathematical statements. The more problems it has successfully solved, the better AlphaProof has become at tackling problems of increasing complexity.

Although AlphaProof was trained to tackle problems across a wide range of mathematical topics, AlphaGeometry 2 - an improved version of a system AlphaGeometry that Google DeepMind announced in January this year-was optimized to tackle problems relating to movements of objects and equations involving angles, ratios, and distances. Because it was trained on significantly more synthetic data than its predecessor, it was able to take on much more challenging geometry questions.

To test the systems' capabilities, Google DeepMind researchers tasked them with solving the six problems given to humans competing in this year's IMO and proving that the answers were correct. AlphaProof solved two algebra problems and one number theory problem, one of which was the competition's hardest. AlphaGeometry 2 successfully solved a geometry question, but two questions on combinatorics (an area of maths focused on counting and arranging objects) were left unsolved.

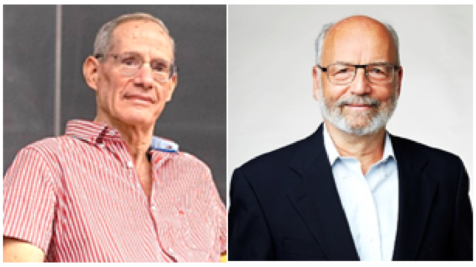
Two renowned mathematicians, *Tim Gowers* and *Joseph Myers*, checked the systems' submissions. They awarded each of their four correct answers full marks (seven out of seven), giving the systems a total of 28 points out of a maximum of 42. A human participant earning this score would be awarded a silver medal and just miss out on gold, the cut-off for which was 29 points.

This is the first time any AI system has been able to achieve a medal-level performance on IMO

questions. Creating AI systems that can solve more challenging mathematics problems could pave the way for exciting human-AI collaborations, helping mathematicians to both solve and invent new kinds of problems. This in turn could help us learn more about how we humans tackle maths. There is still much we do not know about how humans solve complex mathematics problems.
Source: *https://www.technologyreview.com/2024/07/25/1095315/google-deepminds-ai- systems-can-now- solve-complex-math-problems/*

3.8 Awards

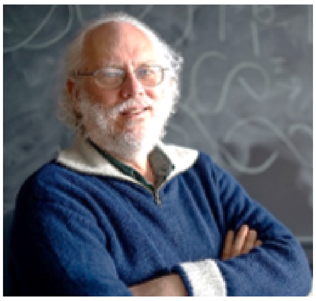
3.8.1 Noga Alon and Adi Shamir Receive the 2024 Wolf Prize in Mathematics



Noga Alon of Princeton University (left) and *Adi Shamir* of the Weizmann Institute of Science in Israel (right) jointly receive the 2024 Wolf Prize in Mathematics. Noga Alon received the Prize "for his fundamental contributions to Combinatorics and Theoretical Computer Science". Adi Shamir received the Prize "for his fundamental contributions to Mathematical Cryptography".
Noga Alon, a professor of applied and computational mathematics, is one of the most prolific mathematicians in the world. His research and developments created new concepts and original methods, and contributed greatly to the development of theoretical research and their applications in discrete mathematics, information theory, graph theory, and their uses in computer science. He has published more than 850 papers, including contributions to biology, economics and neuroscience.
The Wolf Prize citation called out Alon's "profound impact on discrete mathematics and related areas. His seminal contributions include the development of ingenious techniques in combinatorics, graph theory and theoretical computer science, and the solution of long-standing problems in these fields as well as in analytical number theory, combinatorial geometry, and information theory".
Adi Shamir is one of the most senior computer scientists globally. He is a top expert in the fields of information encryption and decryption. Shamir was among the developers of the RSA method that changed the face of computer communication in the world, and was a fundamental pillar in electronic commerce and information security. Shamir's ingenuity extended to TV broadcast encryption, allowing encrypted transmissions exclusively for paying recipients. In recent years, his research delved into T-functions, intricate mathematical tools for information encryption. Shamir's impact also extends to exposing vulnerabilities in encryption systems, developing general mathematical methods for attacks, and pioneering Side Channel Attacks on hardware and software implementations.
Adi Shamir is awarded the Wolf Prize for being a truly exceptional scientist. He has been the leading force in transforming cryptography into a scientific discipline that is heavily based on Mathematics.
His foundational discoveries combine mathematical ingenuity with a range of analytical tools. They had a huge impact on several mathematical areas, advancing both mathematics and society in an unparalleled manner.
The Wolf Foundation has been awarding the international Wolf Prize since 1978 to outstanding scientists and artists from around the world for achievements in the interest of mankind and friendly relations amongst peoples in the categories of Mathematics, Chemistry, Medicine, Agriculture, Physics and Arts.
The Wolf Prize in Mathematics is the third most prestigious international academic award in mathematics, after the Abel Prize and the Fields Medal. Until the establishment of the Abel Prize, it was perhaps the closest equivalent of a "Nobel Prize in Mathematics", since the Fields Medal is awarded every four years only to mathematicians under the age of 40.

Sources:
1. *https://www.princeton.edu/news/2024/07/09/ noga-alon- receives-2024- wolf- prize -mathematics*
2. *https://wolffund.org.il/adi-shamir/*
3. *https://wolffund.org.il/noga-alon/*

3.8.2 Peter Shor of MIT Receives 2025 Claude E. Shannon Award



The IEEE Information Theory Society announced that *Peter Shor*, the Morss Professor of Applied Mathematics at the Massachusetts Institute of Technology (MIT), is the recipient of the 2025 Claude E. Shannon Award. The award recognizes Shor's consistent and profound contributions to the field of information theory.

Renowned for his pioneering work in quantum computation, Shor devised the algorithm known after him, a quantum algorithm that factors integers exponentially faster than any known classical algorithm. His contributions have significantly advanced theoretical computer science, focusing on algorithms, quantum computing, computational geometry and combinatorics. He received the Nevanlinna Prize in the Berlin ICM in 1998.

Shor's academic journey began with a B.A. in mathematics from Caltech in 1981. He earned his Ph.D. in applied mathematics from MIT in 1985 under the guidance of Tom Leighton. Following a postdoctoral fellowship at the Mathematical Sciences Research Institute (MSRI), he joined AT&T's research staff, where he worked from 1986 to 2003. In 2003, Shor returned to MIT as a full professor in the applied mathematics department.

Source: *https://thequantuminsider.com/2024/07/15/mits-peter-shor- receives- 2025- claude -e-shannon -award/*

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4. Obituary Notes

4.1 Professor E. Sampathkumar (10 June 1936 - 11 August 2024)

It is difficult to think that Professor E. Sampathkumar, affectionately called ES by his friends and admirers, is no more with us. Apparently, ES developed some breathing problem on August 9, 2024 when he was at Mysore. He was rushed to the hospital by his friends. But then, he was not responding to the treatment and he passed away on August 11, 2024. His body was then brought down to Bangalore and the last rites were performed on August 12, 2024 by his two sons at his Bangalore residence. Though ES is no more with us, his academic involvements would keep haunting our minds for many more years to come.
ES was born on 10 June, 1936 in Dodda Mallur village of Channapatna taluk, Ramanagara District. He completed his Master's in the Central College, Bangalore in 1955 and his Ph.D. in 1965 at the Karnataka University, Dharwad under the guidance Prof. C. N. Srinivasa Iyengar, who had obtained his D.Sc. degree from Calcutta University. ES was working as Lecturer in this department since 1960. In 1966, he was elevated as Professor in the same department and he continued in this position till 1988. He then shifted to the University of Mysore and finally retired from service during 1996.

Academic Works of ES: Graph Theory was the main area of research of ES. In fact, it is often said that he was the first to teach Graph Theory in India (at Karnataka University) in 1970. The main areas of his research in graph theory were: Domination in Graphs and Semi-graphs. In Domination, he introduced several new parameters: connected, global, strong, weak dominations besides domination balance. In fact, Dr. B. D. Acharya and Dr. H. B. Walikar who were co-workers of ES (besides others, of course) joined with ES and brought out the MRI (Mehta Research Institute, Allahabad) Lecture Notes # 1 in 1979 highlighting the major results in Graph Domination thus far available.
He also introduced several (semi)graph parameters: Degree Sequences, planarity, duals, complete and bipartite semi-graphs, matchings, domination, coloring parameters, topological spaces associated with semi-graphs/ directed semi-graphs. Again, a major part of the above concepts in semi-graphs have been brought out by ES in a book form as an ADMA publication in 2019, which he had edited jointly with Professors C. M. Deshpande, B. Y. Ram, L. Pushpalatha and V. Swaminathan.

My association with ES: My association with ES dates back to 1972 when Professors S. S. Shrikhande and C. R. Rao jointly organized an international conference relating to Graph Theory and Combinatorics at the INSA Building in New Delhi. We were both invited speakers at this conference. The contact that we made at that conference brought us closer in the years to come, to embark upon several joint ventures by way of organizing several conferences and workshops, both national and international, in Discrete Mathematics and, in particular, in Graph Theory. One of the important workshops amongst these was the 8-day workshop on Graph Theory organized by me at the Annamalai University. ES was particularly happy about this workshop in which lecture notes were distributed on the first day of the workshop itself.

Birth of the Ramanujan Mathematical Society: The 50$^{\text{th}}$ annual conference of the Indian Mathematical Society (IMS) was held at Sardar Patel University, Vallabh Vidyanagar, in Anand district of Gujarat, during December 1984. On the second day of the conference, during lunch break, Professor K. S. Padmanabhan (KSP) of the Institute for Advanced Study in Mathematics (RIM in brief) collected a few of the participants of the conference: Professors R. Balakrishnan (myself), ES, G. Rangan, M. S. Rangachari, M. A. Sundaram, T. Soundararajan and a few others (whose names I do not recall now) and projected his view that the country like India, with its size and stature, needed yet another good mathematics journal that aimed at both quality and regularity. Everyone seemed to agree to his suggestion. But then many opined that to start such a journal, there should be a base – a launching pad - and hence there was an imperative need for founding a new mathematics society. As the afternoon session of the conference was about to commence, the discussion was ended at that point and it was decided to continue it in the evening at the end of the afternoon session of the conference. Nearly half of those present in the lunch-break deliberations did not show up in the evening, but that did not deter KSP from his efforts. He continued with his deliberations in which myself, ES, Jayasankaran, M. A. Sundaram and a few others who were present.

Keeping in tune with the discussions, I was requested to call upon a 'Promoters' Meeting'. This I did in 1985 at the National College Campus, Tiruchirappalli in which Professors KSP, ES, G. Sankaranarayanan (GS), Jayasankaran, Ramabhadran and several others were present (besides myself, of course). RMS was officially launched at this meeting with GS as the first President, myself as the first Secretary, ES as the first Academic Secretary and Jayasankaran as Treasurer. Soon I prepared the bye-laws of the Society with the assistance of Professor V. Thangaraj of RIM (who was then at REC, Trichy) and got the Society registered at Tiruchirappalli. I would also like to mention at this point that some people did not exhibit much interest in RMS in the earlier period of the Society as they thought that RMS had been founded as a rival to the Indian Mathematical Society (IMS). Prof. KSP belied that supposition by being the Honorary Librarian of the IMS Library at RIM not only during his tenure as Director of RIM but also even after his retirement from RIM.

Ten years rolled on. In 1996, I organized an International Conference on Discrete Mathematics and Number Theory at Tiruchirappalli. The Number Theory part was organized by Professors Michel Waldschmidt of the University of Paris, Paris VI and Professor Kumar Murty (KM) of the University of Toronto, Canada.

ES somehow impressed upon KM and persuaded him to take up the post of Editor-in-Chief of the Journal of the Ramanujan Mathematical Society (JRMS). This KM did the next year, namely, 1997 and ES himself had taken up the Managing Editorship of JRMS. Since that time, the visibility of JRMS as a leading journal in mathematics was getting increased steadily. ES continued as the Managing Editor of JRMS almost till his death, and needless to state that today JRMS has become one of the leading math journals.

Birth of the Academy of Discrete Mathematics and Applications (ADMA): With the activities in Discrete Mathematics, especially in Graph Theory, started growing, (the late) Dr. B. D. Acharya of the Department of Science and Technology, Government of India started spearheading the move, with the active support of Prof. H. B. Walikar, ES and many other leading discrete mathematicians of our country to establish a society for discrete mathematics. Finally, this came to fruition and the Academy of Discrete Mathematics and Applications (ADMA) was launched in year 2005 at the time of the first Graph Theory Day, namely, June 10, 2005. Naturally ES was made first President of ADMA. Just as RMS launched its journal JRMS, ADMA launched its journal, IJDM (The Indian Journal of Discrete Mathematics and Applications) in the year 2015. Prof ES and Prof. S. Arumugam were the Editors-in-Chief from 2015 to 2018. Like RMS, ADMA too started organizing conferences and workshops. Its annual conferences are usually held around June 10$^{\text{th}}$ (the birth day of ES) of every year and one full session of June 10$^{\text{th}}$ is held in his honour. Incidentally, IJDM is now one of the journals under UGC CARE List. As we could all see, the impact of ES on both RMS and ADMA had been profound.

It may also be mentioned here that Prof. Sampathkumar was a supporter and an Executive Committee member of The Mathematics Consortium, right from its inception in 2016.

This is, so to say, a succinct summary of the academic life of a sublime personality, namely, ES.

I am tempted to ask like Mark Antony in the Shakespeare's play Julius Caesar: Here was an ES. When comes such another?

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4.2 Prof. R. C. Gupta (14 August 1935 – 5 September 2024)

The legendary Indian historian of mathematics, Prof. Radha Charan Gupta passed away in Jhansi on 5 September 2024.

He was born on 14 August 1935 in Jhansi on the Raksha Bandhan day, though the date was given as 26 October 1935 in official records. He had his early education in Jhansi, passing the Intermediate Science examination in 1953. He did his B.Sc. (with physics, chemistry and mathematics) in 1955, and M.Sc. in mathematics in 1957, both from Lucknow University; he was awarded the Gold medal for being topper in his M.Sc. batch. Interestingly, he had also secured a Gold medal for being an outstanding sportsperson (mainly gymnastics) in 1956. He taught in the Lucknow Christian College for a year before moving to the Birla Institute of Technology, Ranchi in 1958, and continued teaching in the mathematics department there till October 1995, when he retired as a professor.

He became interested in the history of Indian mathematics around 1960, influenced by the celebrated book on Hindu mathematics by Bibhutibhushan Datta and Avadhesh Narayan Singh. As he remarked in an interview in 2019 [1], "the book which we now find to be a standard book was not favourably reviewed (by foreign scholars) at that time. So, I decided to devote my life to studying the history of mathematics because at that point in time there was no use asking who was right or wrong". At that time T. A. Saraswati Amma, from Kerala, following her doctoral work on "Geometry in Ancient and Medieval India" carried out earlier at the University of Madras, had submitted her Ph.D. thesis to the Ranchi university in January 1963 [2], and was teaching Sanskrit at a College in Ranchi. R. C. Gupta came in contact with her and did his Ph.D. under her supervision. As Arithmetic and Algebra had been covered in the book by Datta and Singh, and Saraswati Amma had worked on geometry herself, it was decided that Gupta would work on trigonometry in ancient and medieval India [3].

Gupta did not know much Sanskrit when he began his research, but acquired proficiency in the language all by himself, so much so that he could translate many untranslated stanzas in the works of relevance for him, and was able to point out errors in earlier interpretations in some cases, and provide corrections. Gupta's Ph.D. thesis "Trigonometry in Ancient and Medieval India" was submitted to the Ranchi University in 1970, and he was awarded the degree in 1971. The range and depth of his scholarship can be gauged from the fact that the analyses and conclusions in his very remarkable thesis were based on 545 references to books and articles. Now, the relation between a chord and the corresponding arc in a circle was the central feature of Greek trigonometry. For applications in trigonometry, one had to constantly refer to tables of chords for conversion of arcs to chords, and vice versa. In contrast to this, Indians dealt with the half-chords, and the Indian *jyā* or *jīvā*, $R \sin \theta$ corresponding to an arc $R\,\theta$, or the angle θ, is the modern sine except for a factor R, where R is the radius of the circle pertaining to the arc. It is considered as a function (of θ). This enabled Indians to give algorithms for computing various astronomical quantities (in the verse form) in terms of the sine and cosine functions, something lacking in Greek astronomy. Gupta's thesis is a very comprehensive account of the evolution of trigonometry in India, right from its introduction in *Āryabhaṭīya* (499 CE) (including methods to find the *jyā*, $R \sin \theta$ for any θ) up to the 17$^{\text{th}}$ century, including the discussion of the infinite series for the sine and cosine functions in the Kerala school. The thesis also discusses the influence of Indian trigonometry abroad, and

[1] R. C. Gupta in conversation with C. S. Aravinda, Bhāvanā- The mathematics magazine, October 2019.

[2] The thesis was later published as a book, with the same title, by Motilal Banarsidass in 1979, and a revised edition was brought out in 1999; the book has served as a major resource on the topic.

[3] Datta and Singh had made notes on various topics in Indian mathematics apart from arithmetic and algebra, including trigonometry, and R. C. Gupta refers to these in his thesis. K.S. Shukla of Lucknow university edited and revised them later, and a series of articles based on them were published in the *Indian Journal of History of Science* from 1983 onwards with Datta, Singh and Shukla as the authors.

also foreign influence on it in the later stages.
The *Mahābhaskarīya* of Bhāskara-I (around 600 CE) contains a simple and elegant formula for approximating the sine function:

$$\sin\,\theta = \frac{4\theta(180-\theta)}{40500-\theta(180-\theta)} (\theta \text{ in degrees}).$$

No justification had been provided in the original, or in other earlier literature. In a paper published in 1967, even before the completion of his thesis, Gupta provided three derivations of the formula based on reasonable assumptions. The error in the value of $\sin\,\theta$ calculated from the formula is less than 1% except when the values are very small.
R. C. Gupta wrote on the various second order interpolations in the Indian texts, and their usefulness. "Solution of the astronomical triangle as found in *Tantrasaṅgraha*" is an important paper (1974). Here, 10 rules in spherical trigonometry pertaining to the diurnal motion of objects on the celestial sphere given by Nīlakaṇṭha Somayājī in *Tantrasaṅgraha* (1500 CE) are derived. Another important paper (1987) pertaining to spherical trigonometry in India is "Mādhava's Rule for finding the Angle between the Ecliptic and the Horizon and Āryabhaṭa's knowledge of it".
Gupta's "Spread and Triumph of Indian Numerals" (1983) is indeed a classic. It describes the dynamics of the spread of Indian numerals throughout the world, especially in China, Islamic/Arab regions, south-east Asia, and Europe. Concluding the paper, he remarks "This was, no doubt, the victory of Indian culture, but it was also a victory for the mind of man, who finally, in the long history of written numerals, arrived at a mature, abstract decimal place-value notation".
Gupta's book "Ancient Jaina Mathematics" is a very important work, where he elaborates on many distinctively Jaina contributions to mathematics. It includes a discussion of ancient Jaina cosmography and related calculations in a Jaina work, *Tilyopaṇṇatti* which involves not only sums of geometrical progression but examples of perfect numbers [4]. It seems that the author of *Tilyopaṇṇatti* was aware of the fact that $P_n = (2^n-1)2^{n-1}$ (integer n) are perfect numbers, when 2^n-1 is a prime. Concluding the discussion, he remarks that "We can say almost safely that all perfect numbers represent the number of *khaṇḍas* (parts) of some island or sea in Jaina cosmography in which the number of rings of such islands and seas are stated to be *asaṁkhyāta* ('unenumerable')". The book also discusses (i) the (approximate) length of an arc, s in terms of its associated chord, c and height, $h : s = \sqrt{c^2+6h^2}$, (ii) area, A of a segment of height h between an arc and the chord of length c in a circle : $A = \sqrt{10}\frac{ch}{4} \approx 3\frac{ch}{4}$, (iii) surface area, A of a spherical segment: $A = \frac{p}{4}s = \frac{\pi c}{4}s$, where p and c are the perimeter and the chord associated with the circular base of the segment, and s is the arc-length associated with it. Mahavīrācārya's formula for the area of a spherical segment in (iii) is actually, $A = \frac{p}{4}\times$ *viṣkambha*, and it is Gupta who argues convincingly that the *viṣkambha* of the segment should be identified with the arc-length s. For the other two formulae in (i) and (ii), the Jaina value of $\pi = \sqrt{10}$ is implicit; this value is used in other Indian texts also. Carrying out computations using a pocket calculator, Gupta found that the optimal expression for s in (i) is $s = \sqrt{c^2+5.8h^2}$, and in (ii), the formula $A = 3\frac{ch}{4}$ is the simplest and best in its category.
Elsewhere in the book, Gupta discusses the formula $P(a,b) = \sqrt{16a^2+24b^2}$ for the perimeter of an ellipse with the semi-major axis, a, and semi-minor axis, b, given by Mahāvīrācārya, which is of course approximate, and remarks that "Mahāvīra deserves the credit of devising the first approximate results to calculate the perimeter of an ellipse".
Some of his other important papers are "Parameśvara's Rule for the Circumradius of a Cyclic Quadrilateral", "An Indian form of Third Order Taylor Series Approximation of the Sine", "The Chronic problem of Ancient Indian Chronology", "Early Pandiagonal Magic Squares in India ", "Yantras and Mystic diagrams: A wide area of Study in Ancient and Medieval Indian Mathematics", and many papers on Indian Combinatorics including "The Last Combinatorial Problem in Bhāskara's *Līlāvatī*". The *Līlāvatī* problem concerns the rule for the number of ways, N

[4]A positive integer is a perfect number if it is the sum of its proper divisors.

in which the digits in an n-digit number can be arranged such that the sum of the digits is $S = n + m, (m < 9)$, where each digit can be any number from 1 to 9. The rule states that $N = \frac{(n+m-1)!}{(n-1)!m!}$. None of the commentaries on *Līlāvatī* explains this rule. Gupta's derivation of the result is in the same spirit as that of some earlier scholars, but he extends the problem to one in which there is no restriction on m. At the end of the paper, he quotes a western scholar Sharon Kunoff who remarked that "the algebra of combinatorics in the 12$^{\text{th}}$ century and perhaps earlier was considerably more advanced in Hindu mathematics than in western circles".

Under the 'Read and Learn Plan' of the National Council of Educational Research and Training (NCERT) in India, several books on scientific topics for school students were published in late 1990's. The aim was to encourage school students to learn various important and significant aspects of Indian science and mathematics, apart from the regular syllabus topics. R. C. Gupta wrote two wonderful books in Hindi, under the plan : '*Prāchīn Bhāratīya Gaṇit kī Aitihāsik va Sāṁskṛtik Zhalakiyān*' (Historical and Cultural Glimpses of Ancient Indian mathematics), and '*Madhyakālīn Bhāratīya Gaṇit kī Aitihāsik va Sāṁskṛtik Zhalakiyān*' (Historical and Cultural Glimpses of Medieval Indian mathematics). In these books, Gupta explains several important aspects of Indian mathematics in a simple, and enjoyable manner without diluting the topics, in a chatty and student-friendly style, with historical details. The second book contains discussions of the important later developments like the infinite series for π, and sine and cosine functions in the Kerala school of mathematics, and a moving account of Srinivasa Ramanujan's life with interesting anecdotes, and his mathematical achievements. Substantial parts of these books can and should get into regular school textbooks on mathematics for suitable classes, straight away.

R. C. Gupta was known for his meticulous scholarship, thorough study of the literature on any topic, and a critical assessment before finalising an article on it. He never took to computers, and had his own system of data cards for references. He was concerned about the poor quality of work, lack of awareness of current research and also of an international perspective, which was true of many researchers in the field of history of mathematics in India. There is another side to the 'international perspective'. On the other hand, in his earlier works (for example, his Ph.D. thesis), Gupta was critical of the prejudices of some western scholars.

Gupta was one of the founders of the Indian Society for History of Mathematics, and was the editor of its journal *Gaṇita Bhāratī* right from its inception in 1979 till 2005. He edited it during this period with great dedication and distinction. In 1991 he was elected a Fellow of the National Academy of Sciences, Allahabad. He became a corresponding member of the International Academy of History of Sciences in February 1995. He has travelled widely in India and abroad. In 2009 he was awarded the Kenneth O. May prize by the international Commission on the History of Mathematics - the highest honour for contributions to history of mathematics - alongside the British mathematician Ivor Grattan-Guiness of UK. He has been the only Indian to receive this prize so far. In 2023, he was awarded the Padma Shri by the Government of India, for his contributions in the field of literature and education.

Prof. R. C. Gupta had a warm personality, and was encouraging to younger researchers in the field of history of mathematics in India, by offering very professional advice, and allowing access to his vast collection of books, journals and articles. He supported endowment lectures in the names of Sarasvati Amma at the Kerala Mathematical Association, K. V. Sarma at the Prof. K. V. Sarma Research Foundation, Chennai, K. S. Shukla at the National Academy of Sciences, Allahabad, and some more, as far as I can recollect, through generous grants.

The history of mathematics community in India and abroad will miss Prof. R. C. Gupta!

In preparing this article I have benefited substantially by consulting the following edited volume, and the two articles:

1. *Gaṇitānanda* - Selected works of Radha Charan Gupta on History of Mathematics, Ed. By K. Ramasubramanian, Indian Society for History of Mathematics, New Delhi, 2015.

2. C. S. Aravinda, 'Gaṇitānand: A Different History Tale from Jhansi - R. C. GUPTA in

conversation with C. S. Aravinda', Bhāvanā - *The mathematics magazine*, pp. 22-31, October 2019, Bengaluru (Online access: bhavana.org.in).

3. S. G. Dani, '*Gaṇitānanda* - Selected Works of Radha Charan Gupta on History of Mathematics-A review', Bhāvanā - *The mathematics magazine*, pp. 32-44, October 2019, Bengaluru.

More details on the life and works of R. C. Gupta and references are available in these three sources.

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4.3 Prof. Jagannath N. Salunke (30 June 1955 - 28 July 2024)

It is with deep sadness that we announce the passing away of Professor J. N. Salunke on 28th July 2024, at the age of 69. Born on 30th June 1955, Professor Salunke dedicated his life to the study and teaching of mathematics, leaving a profound impact on those who had the privilege of learning from and working with him.

Professor Salunke spent the majority of his career at the School of Mathematical Sciences, Swami Ramanand Teerth Marathwada University (SRTM), Nanded, from where he retired in 2015. He also served at the School of Mathematical Sciences, North Maharashtra University, Jalgaon, from 2007 to 2011. His dedication to the field was evident in the time he devoted to his students and his passion for research.

During his academic career, Professor Salunke guided 13 M.Phil. and 12 Ph.D. students, playing a key role in their intellectual development. He was the author/co-author of 25 books covering a range of topics including algebra, analysis, and differential equations, and he published around 115 research papers.

Professor Salunke was an active member of The Mathematics Consortium, and The Marathwada Mathematical Society, frequently engaging with problems posed in the TMC Bulletin and the Bulletin of the Marathwada Mathematical Society. His love for mathematics and problem-solving was a defining feature of his life.

He will be remembered as a dedicated teacher, a thoughtful mentor, and a humble scholar. His contributions to the field of mathematics and his influence on the lives of his students and colleagues will endure. Our hearty tributes to Prof. Salunke.

Usha Sangale
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5. A Peep into History of Mathematics

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Beginning with this issue we will be carrying a column introducing our readers to some recent works in the area of history of mathematics through brief reviews of selected papers, on various topics in the area. The readers are encouraged to contribute individual entries of interest in the format seen here, in about 300 words each. - Managing Editor

Lloyd Strickland, Why did Thomas Harriot invent binary? Math. Intelligencer 46 (2024), no. 1, 57-62.

The history of usage of binary numeration and arithmetic was traced in the modern context initially to Gottfried Leibniz (1646-1716) and then further back to an English mathematician, astronomer and alchemist Thomas Harriot (1560-1621), about a century earlier. This short paper explores "how and when" Harriot invented the binary numeration. The author relates it to certain weighing experiments that Harriot carried out between 1601 and 1605, to determine the weights of different substances, mainly a variety of wines, in a measuring glass. Attention is drawn especially to one particular manuscript, which has been reproduced with the article, which contains a record of a weighing experiment at the top, and examples of binary notation and arithmetic at the bottom.

Michael Friedman, A tale of a threshing machine: images of the Voigt-Leibniz mathematical-agricultural machine at the beginning of the 18th century, Stud. Hist. Philos. Sci. 105 (2024), 17-31.

A book with the (Latin) title *Deliciae physico-mathematicae* (Physico-mathematical delights), was published by Georg Philipp Harsdeörffer, in 1651, in which he discusses, among other things a machine involving a unique mechanism based on a pinned drum. While it is known that such a mechanism was used in making musical instruments, the book suggests that it was also used by weavers and threshers. More interestingly the title of the book suggests that the machines were considered *mathematical*, and presumably involved mathematicians in their construction. While there are no worthwhile details available about the machines that Harsdeörffer meant, there are documentary records of a threshing machine that was developed and improved by Jobst Heinrich Voigt and Gottfried Wilhelm Leibniz between 1699 and 1700. The present article discusses, in the context of the machine, the close interaction between mathematics and agriculture, in 18$^{\text{th}}$ century Europe, referring to numerous works from that time, including several papers of Leibniz.

Ian Stewart, Galois and the simple group of order 60, Arch. Hist. Exact Sci. 78 (2024), no. 1, 1-28.

It is well-known that in a letter that Évariste Galois wrote to his friend Auguste Chevalier, on the night before the fatal duel, he shared many of his findings, which later developed into what is now called Galois theory. The same letter contains also a passing remark to the effect that 60 is the smallest possible order for a non-cyclic simple group. There is no proof of the statement found in his surviving papers, including any scraps, as the author puts it. Today one would derive this statement as a simple consequence of Sylow's theorem, a deep result in basic group theory, that was published in 1872, some 40 years after Galois died. In this context it is generally doubted whether Galois had a (valid) proof of the statement. In particular, Peter Neumann in his biographical book *The Mathematical Writings* of *Évariste Galois* asks "How could he have excluded orders such as 30, 32, 36, 40, 48, 56?" (based on the knowledge of group theory at that time). This article discusses the group-theoretic issues involved, in the framework around them at that time, and concludes that while we can never be sure whether Galois had a proof of it or not, it is possible to prove the statement by methods that Galois is known to have possessed, or probably possessed.

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6. Book Review

The book: A Course in Linear Algebra by Raju K. George, Abhijit Ajaykumar
(University Texts in Mathematical Sciences, Springer Nature (2024))
Review by
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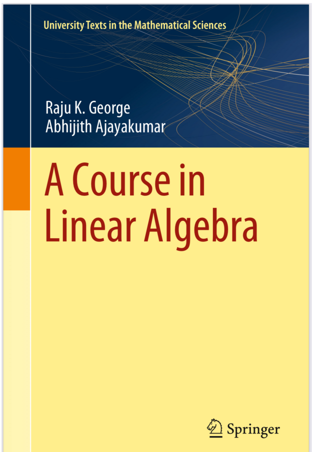
Linear Algebra is one of the essential mathematical disciplines that undergraduate science and engineering students need for their studies. This book aims to present an introduction to linear algebra and basic functional analysis in a simple manner and with problem solving approach, that will be helpful for a wide section of students and teachers.



The book contains 13 chapters, of which the first six chapters deal with the basic theory of linear algebra and basic functional analysis, Chapter 7 provides various real life problems where tools of Linear Algebra and Functional analysis are used for problem solving and Chapters 8-13 contain a good collection of problems along with solutions.

Chapter 1 contains elementary set theory, metric spaces and matrix theory which are foundations for linear algebra and functional analysis study. Solutions to the system of linear equations are also discussed. Chapter 2 introduces the primary concept of Linear Algebra, namely, Vector Spaces along with numerous examples. Important notions of subspaces, linear dependence, basis, and dimension are discussed in detail.

Chapter 3 focusses on Linear transformations. Range space, null space, rank, nullity and related concepts. After introducing matrix representations of linear transformations most of the abstract concepts related to linear transformations are dealt with in terms of matrices. The algebra of linear transformations, change of coordinate matrix, linear functionals, dual space, etc., are also discussed. A geometrical overview of varied linear transformations in R^2 is presented which would kindle the reader's geometric intuition. Chapter 4 contains the spectral analysis of matrices. Here, linear operators in terms of matrices, eigenvalues, eigenvectors, and some classes of polynomials satisfied by matrices are defined. Important theorems like the Cayley-Hamiton theorem, Schur triangularization theorem are presented. Diagonalization, generalized eigenvectors and Jordan- canonical form are also studied in detail.

In Chapter 5, norms on vector spaces are defined and properties of normed linear spaces are discussed in detail. Inner product space is defined and basic notions and theorems on inner product are proved, followed by a discussion on orthonormal sets, orthogonal projection and Gram-Schmidt Orthonormalization. Completeness of abstract spaces, Banach and Hilbert spaces are explained with many examples. In short, this chapter briefly introduces fundamental ideas of functional analysis. Chapter 6 gives a flavor of operator theory by discussing bounded linear maps and their properties. Fundamental theorems on the adjoint operator, self- adjoint operators, normal operators, unitary operators are proved. Singular Value Decomposition (SVD) and pseudo-inverse of matrices are discussed in detail. This chapter ends with a discussion on the least square solutions of system of linear equations.

Chapter 7 gives application of tools of Linear algebra and functional analysis to Cryptography, Satellite control problems, control theory and Artificial neural networks.

Each chapter is provided with an ample number of examples and exercise questions. Solutions to selected exercise questions are given at the end. Chapters 8 to 13 contain more than 500 solved problems of varying difficulty levels based on the topics discussed in Chapters 1-7. Detailed solutions are given for each question to provide a better understanding of the ideas discussed. These chapters will help all students, especially those who are attempting competitive exams, for studies to master the subject and attain problem-solving skills.

The book aims mainly at graduate and engineering students and can be used as a primary text in a suitable course in Linear Algebra or as a supplementary reading. Abstract concepts are dealt with comparatively less rigor, keeping in mind a first-time reader. This feature of the book would help students familiarize themselves with the subject matter faster and stimulate further interest in the subject.

In summary 'A Course in Linear Algebra', published by Springer in 2024, is an excellent addition to the subject Linear Algebra.

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7. International Calendar of Mathematics Events

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January 2025

- January 8-11, 2025, 2025 Joint Mathematics Meetings (JMM), Seattle, Washington. *jointmathematicsmeetings.org/jmm*
- January 12-15, 2025, ACM-SIAM Symposium on Discrete Algorithms (SODA25), Astor Crowne Plaza, New Orleans French Quarter, New Orleans, Louisiana, US. *www.siam.org/conferences/cm/conference/soda25*
- January 23-24, 2025,Connections Workshop: Probability and Statistics of Discrete Structures, SLMath, 17 Gauss Way, Berkeley, CA 94720. *www.slmath.org/workshops/1085#overview_workshop*

February 2025

- February 5-7, 2025, International Conference on Pure Mathematics and its Applications (ICPMA 2025), Department of Pure Mathematics, University of Calcutta, Kolkata, 700019. *sites.google.com/caluniv.ac.in/icpma2025*
- February 6-7, 2025, Connections Workshop: Extremal Combinatorics, SLMath 17 Gauss Way, Berkeley, CA 94720 US. *www.slmath.org/workshops/1087#overview_workshop*
- February 10-12, 2025, Unhacked an International Interdisciplinary Conference on Cybercrimes and Cyberattacks, Vienna, Austria. *momentera.org/conferences/unhacked-conference/*
- February 10-14, 2025, Introductory Workshop - Graph Theory: Extremal, Probabilistic and Structural, SLMath, 17 Gauss Way, Berkeley CA 94720. *www.msri.org/workshops/1088*
- February 10-14, 2025, ACD2025 - Computational Interactions between Algebra, Combinatorics, and Discrete Geometry, Los Angeles, CA, United States. *https://www.ipam.ucla.edu/ programs/workshops/ computational-interactions-between- algebra-combinatorics-and-discrete-geometry/*
- February 28 - March 2, 2025, WALCOM 2025 - 19th International Conference and Workshops on Algorithms and Computation, Chengdu, China. *https://tcsuestc.com/walcom2025/*

March 2025

- March 2-7, 2025, SIAM Conference on Computational Science and Engineering (CSE25), Fort Worth Convention Center, Fort Worth, Texas, US. *www.siam.org/conferences/cm/conference/cse25*
- March 3-7, 2025, AIM Workshop: The Geometry of Polynomials in Combinatorics and Sampling American Institute of Mathematics, Pasadena, California. *aimath.org/workshops/upcoming/geompolysampling/*

- March 17-21, 2025, Algebraic and Analytic Methods in Combinatorics, SLMath, 17 Gauss Way, Berkeley, CA 94609 US. *www.slmath.org/workshops/1091#overview_workshop*
- March 17-21, 2025, AIM Workshop: All Roads to The KPZ Universality Class American Institute Of Mathematics, Pasadena, California. *aimath.org/workshops/upcoming/roadtokpz/*
- March 17-21, 2025, The Legacy of John Tate and Beyond, Harvard University Science Centre, MA 02138, US. *www.math.harvard.edu/event/the-legacy-of-john-tate-and-beyond/*
- March 22-23, 2025, Mid-Atlantic Topology Conference 2025, University of Virginia Charlottesville, Virginia. *https://sites.google.com/view/matc-25*
- March 31 - April 4, 2025, AIM Workshop: New Directions In G2 Geometry, American Institute of Mathematics, Pasadena, California. *aimath.org/workshops/upcoming/g2geometry/*

April 2025

- April 7-11, 2025, Interactions Between Harmonic Analysis, Homogeneous Dynamics, and Number Theory, SLMath, 17 Gauss Way, Berkeley, CA 94720 USA. *www.slmath.org/workshops/1089#overview_workshop*
- April 14-18, 2025, AIM Workshop: Integro-Differential Equations in Many-Particle Interacting Systems, American Institute Of Mathematics, Pasadena, California. *aimath.org/workshops/upcoming/manyparticle/*
- April 21-25, 2025, Detection, Estimation, and Reconstruction in Networks, SL Math, 17 Gauss Way, Berkeley, CA 94720 USA. *www.slmath.org/workshops/1098#overview_workshop*
- April 28 - May 2, 2025, AIM Workshop: Moments in Families of L-Functions over Function Fields, American Institute of Mathematics, Pasadena, California. *aimath.org/workshops/upcoming/momentsoverff/*

May 2025

- May 1-3, 2025, SIAM International Conference on Data Mining (SDM25), Westin Alexandria Old Town, Alexandria, Virginia, US. *www.siam.org/conferences/cm/conference/sdm25*
- May 8-10, 2025, International Conference on Non-Linear Analysis and Optimization (with Workshop on Fixed Point Theory and its Applications), Kathmandu University, Dhulikhel, Kavre, Nepal. *icanopt2025.ku.edu.np/*
- May 12-16, 2025. AIM Workshop: Algorithmic Stability: Mathematical Foundations for the Modern Era, American Institute of Mathematics, Pasadena, California. *aimath.org/workshops/upcoming/algostabfoundations*
- May 19-22, 2025, Constructive Functions 2025, Vanderbilt University, Nashville, Tennessee, USA. *my.vanderbilt.edu/constructivefunctions2025/*

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8. Problem Corner

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In the July 2024 issue of TMC Bulletin, we posed two problems, a problem from Number Theory and a problem form Geometry. We have received three solutions of the problem of Number Theory; one by Debabrota Mondal, IIT Bhubaneswar, one by Dr. Vinay Acharya, Fergusson College, Pune and one by the editor of the section. Also we have received one solution of a Number Theory problem posed in October, 2023 issue. This is solved by Prof. J. N. Salunke, Latur, Maharashtra. In this issue we will discuss the solution of Number Theory.
Also, in this issue we pose two problems, one from algebra and one from strategy of games for our readers. Readers are invited to email their solutions to Dr. Udayan Prajapati (ganit.spardha@gmail.com), Coordinator, Problem Corner, before 15$^{\text{th}}$ November, 2024. Most innovative solution will be published in the subsequent issue of the bulletin.

8.1. Problem posed in the previous issue (Dr. Udayan Prajapati, St. Xavier's College, Ahmedabad)

Show that the equation $x^6+x^5+x^4+x^3+x^2+x+1=23456789$ has no integer solution.

Solution (by Debabrota Mondal): The given equation can be simplified as

$$x(x+1)(x^4+x^2+1)+1=23456789 \cdots\cdots (1)$$

A nice observation is $23456789 \equiv 4 \pmod 5$.
If a is a solution of (1) then we have the following cases:

(i) If $a \equiv 0$ or $4 \pmod 5$ then $a(a+1) \equiv 0 \pmod 5$.
So $a(a+1)(a^4+a^2+1)+1 \equiv 1 \pmod 5$.

(ii) If $a \equiv 1$ or $2 \pmod 5$ then $a(a+1)(a^4+a^2+1)+1 \equiv 2 \pmod 5$.

(iii) If $a \equiv 3 \pmod 5$ then $a(a+1)(a^4+a^2+1)+1 \equiv 3(4)(91)+1 \equiv 3 \pmod 5$.

So, from all the cases, there is no integer a such that $a(a+1)(a^4+a^2+1)+1=23456789$.

Second Solution (by Dr. Vinay Acharya): Obviously $x=1$ is not a solution of the equation.
The given equation $x^6+x^5+x^4+x^3+x^2+x+1=23456789 \cdots\cdots (2)$
can be simplified as $\frac{x^7-1}{x-1}=23456789$ i.e. $\frac{x^7-1}{x-1}-1=23456788$.
That is $x^7-x=23456788(x-1) \cdots\cdots (3)$
Now the left hand side of (3) is a multiple of 7. Also the greatest common divisor of 7 and 23456788 is 1. So $x \equiv 1 \pmod 7$. That is $x^6+x^5+x^4+x^3+x^2+x^2+x+1 \equiv 0 \pmod 7$, but the right hand side of (2) is not a multiple of 7.

Third Solution: (by Dr. Udayan Prajapati):
(2) can be simplified as $x(x^3+1)(x^2+x+1)=23456788 \cdots\cdots (4)$
The right side of the equation (4) is $\equiv 1 \pmod 3$.
If $x \equiv 0,1,2 \pmod 3$ then the left hand side of (4) is $\equiv 0 \pmod 3$. So there is no integer solution of the given equation.

8.2. Problem posed in the October 2023 issue (Dr. Vinay Acharya)

5, 5, 6 and 5, 5, 8 are isosceles triangles with integer sides and with equal integer area.
Determine all such pairs of isosceles triangles with integer sides having equal integer area.

Solution (by late Prof Jagannath Salunke):
For $a,b,c \in \mathbb{N}$ with $b \neq c$, let isosceles triangles with sides a,a,b and a,a,c be of the same area.
Then by Heron's formula

$$\sqrt{\left(a+\frac{b}{2}\right)\left(\frac{b}{2}\right)\left(\frac{b}{2}\right)\left(a-\frac{b}{2}\right)}=\sqrt{\left(a+\frac{c}{2}\right)\left(\frac{c}{2}\right)\left(\frac{c}{2}\right)\left(a-\frac{c}{2}\right)}.$$

So, $(2a+b)(2a-b)b^2 = (2a+c)(2a-c)c^2$.
Hence, $4a^2(b^2-c^2) = (b^4-c^4) \to (2a)^2 = (b^2+c^2). \cdots\cdots (*)$
Therefore, $b, c, 2a$ forms a Pythagorean triple and b, c are both odd or both even. But since sum of squares of odd integers is not a perfect square, b, c must be both even, say,

$$b = 2x \text{ and } c = 2y \text{ for some } x, y \in \mathbb{N}. \text{ So by } (*), x^2 + y^2 = a^2$$

Therefore, x, y, a form a Pythagorean triple. Now by Euler's formula, any Pythagorean triple must be of the form $x = 2krt, y = k(r^2-t^2), a = k(r^2+t^2)$ where r, t, k are positive integers with $r > t$, and with r and t coprime and not both odd.
Thus, $a = k(r^2+t^2), b = 4krt$ and $c = 2k(r^2 = t^2)$.
Thus, all pairs of isosceles triangles with integer sides and equal areas given, with the choice of r, t and k as above, are given by
(i) $k(r^2+t^2), k(r^2+t^2), 4krt$ and (ii) $k(r^2+t^2), k(r^2+t^2), 2k(r^2-t^2)$.
It is easy to verify that both such triangles have same area.

Problem for this issue

Let $f(n) = n^3$ for every natural number n.
Let $x = 0.f(1)f(2)f(3)\cdots$. In other words, to obtain the decimal representation of x write the numbers $f(1), f(2), f(3)\cdots$ in base 10 in a row (without leading zeroes). Is x a rational?

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Recent TMC Activity
Short Term Training Program (STTP) on
Mathematical Modelling and Simulation in Physics
Organized by
Parul Institute of Technology, Parul University, Vadodara &
The Mathematics Consortium
13, 14, 20, 21, 22 August, 2024

Left to right: Dr. N. Jhala HOD, Applied scs; Ms. Shivani Shah, STTP Co-ord.;
Dr. V. H. Pradhan, and Dr. Jaysh Dodia, SVNIT, Surat;
Dr. Swapnil Parikh, Principle, PIT; Dr. Kaushal Patel, STTP co-ord.

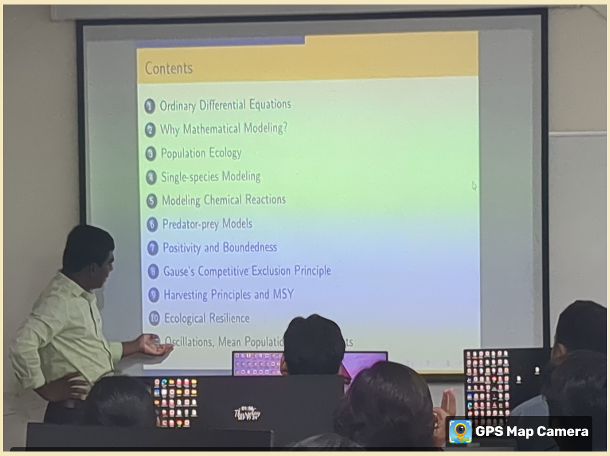
Compact Courses organized by
Professor R. Balakrishanan Endowment Trust (RBET), Tiruchirappalli and The Mathematics Consourtium (TMC) during September 2024

RBET Representatives: Dr. T. Tamizh Chelvam, Dr R. Balakrishnan
TMC Representatives: Dr. Krishnendu Gongopadhyay, Dr S. A. Katre

1. Region: Delhi, Uttar Pradesh and Haryana; Co-ordinators: Dr. Deepti Jain and Dr. Sudhakar Yadav. On: Differential Equation Models in Ecology, September 19-21, 2024, Sri Venkateswara College, University of Delhi, New Delhi.
Speaker: Dr Bapan Ghosh, Associate Professor, IIT Indore.

2. Region: Maharashtra and Gujarat; Co-ordinators: Dr. Chirag M. Barasara. Dr. Kamlesh A. Patel. On: Algebra, September 23-27, 2024, Shri U. P. Arts, Smt. M. G. Panchal Science, and Shri V. L. Shah Commerce College, Pilvai, Gujarat.
Speaker: Dr. Atulkumar V. Kadia, Dept. of Mathematics, Hemchandracharya North Gujarat University.

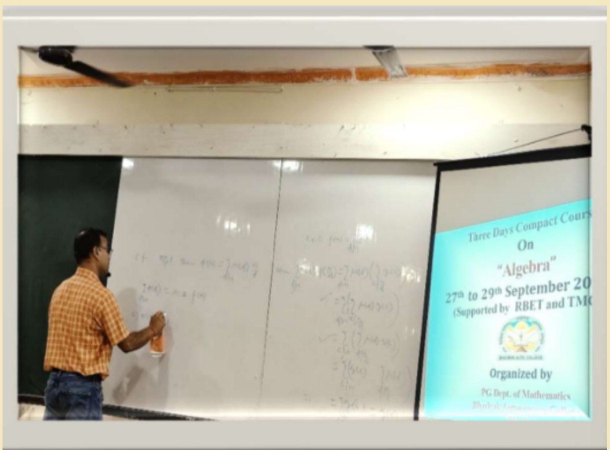
3. Region: Andra Pradesh, Telangana and Orissa; Co-ordinators: Dr. Sasmita Barik and Dr. A. K. Sahoo. On: Algebra, September 27-29, 2024, Bhadrak Autonomous College, Bhadrak, Odisha, 756100.
Speaker: Prof. Brundaban Sahu, NISER Bhubaneswar, Odisha.



Course 1: Speaker Dr. Bapan Ghosh

Course 2: Speaker Dr. Atulkumar Kadia

Course 3 at Bhadrak, Odisha.



Speaker Prof. Brundaban Sahu

Julius Wilhelm Richard Dedekind (06 Oct. 1831 - 12 Feb. 1916)

A German mathematician and philosopher. Made important contributions to abstract algebra (particularly ring theory), algebraic number theory and the axiomatic foundations of arithmetic. Best known contribution is the formal definition of the real numbers through the notion of Dedekind cut. Considered as a pioneer in the development of modern set theory and of the philosophy of mathematics known as logicism.

August Ferdinand Möbius (17 Nov. 1790 - 26 Sept. 1868)

A German mathematician and astronomer. Best known for his discovery of the Möbius strip. Many mathematical concepts are named after him, including the Möbius plane, the Möbius transformations important in projective geometry, the Möbius transform, Möbius function and the Möbius inversion formula in number theory. Introduced homogeneous coordinates into projective geometry and the barycentric coordinate system.

Thomas Jan Stieltjes (29 Dec. 1856 - 31 Dec. 1894)

A French mathematician and Civil Engineer. Worked on almost all branches of Analysis, Continued Fractions and Number Theory. Called 'the father of the "analytic theory of Continued fractions" and awarded the Ormoy Prize of the Academie des Sciences in 1893. Remembered for Stieltjes Integrals. Also worked on divergent series and discontinuous functions and contributed to Ordinary and Partial Differential equations, the gamma functions, interpolation, and Elliptic functions.

Publisher

The Mathematics Consortium (India),
(Reg. no. MAHA/562/2016 /Pune dated 01/04/2016),
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Printers

AUM Copy Point, G-26, Saffron Complex, Nr. Maharana Pratap Chowk,
Fatehgunj, Vadodara-390001; Phone: 0265 2786005;

Annual Subscription for 4 issues (Hard copies)

Individual	:	TMC members: Rs. 800;	Others: Rs. 1200.
Societies/Institutions	:	TMC members: Rs. 1600;	Others: Rs. 2400.
Outside India	:	USD ($) 50.	

The amount should be deposited to the account of
"The Mathematics Consortium", Kotak Mahindra Bank, East Street Branch,
Pune, Maharashtra 411001, INDIA.

Account Number: 9412331450, IFSC Code: KKBK0000721