The Mathematics Consortium

BULLETIN

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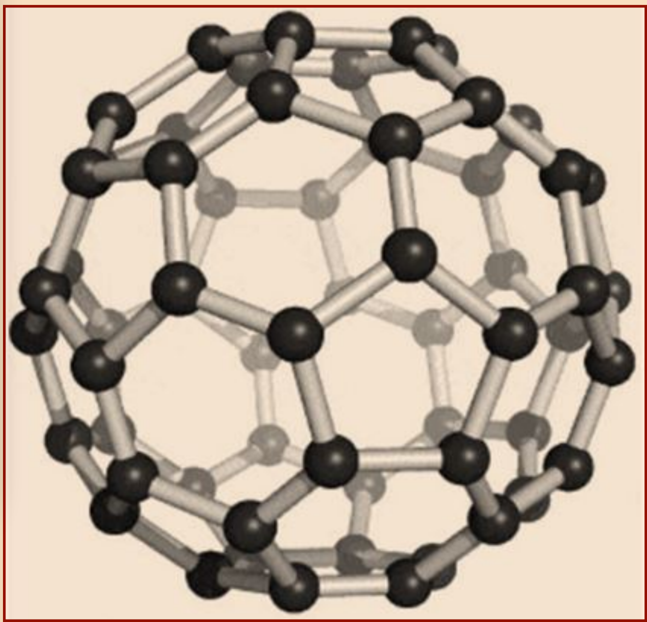
Buckminster fullerene



The Montreal Biosphere

Chief Editor: Ravindra S. Kulkarni Managing Editor: Vijay D. Pathak

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Contents

About the Cover Page:
A class of carbons, called 'fullerenes' were discovered around early 1980s. The second figure on the cover page is a graph model of Buckminster fullerenes C60- molecule of the year 1991, which is named after the American architect Buckminster who designed the Montreal biosphere (first figure) which resembles the model. Fullerene graphs, whose vertices represent carbon atoms and edges represent bonds between atoms, are cubic, three-connected planar graphs with twenty hexagons and twelve pentagons.

Typeset and designed by: Praanjala Typesettings, Contact: +91 99603 03874, Email: mr.texdoc@gmail.com.

From the Editors' Desk

The 21^{st} century has brought new challenges for educational systems as today's "Pre-knowledge requirement" for school children is very different from 3 decades before. Great social changes and improvement of informational sources call for systematic planning of mathematics and science curriculum responding to the social needs of the new age children. National council of teachers of mathematics of America (NCTM) proposed following five skills that students should acquire while in school: Problem solving, Communication, Reasoning, Connection and Representation. To achieve these objectives it is necessary to update the curriculum of Mathematics at school level by including relevant new topics in the curriculum.

According to Principles and Standards for School Mathematics (PSSM), guidelines produced by the NCTM in 2000 for mathematics educator, "Discrete Mathematics and Graph theory should be an integral part of the entire content of the School-Math curriculum". It is believed that teaching of these topics can help the students to acquire three of the five skills mentioned above, namely, Connection, Representation and Problem solving.

The connection standards claim states that, "the Instructional programs at school level should enable all students to recognize and use connections among mathematical ideas and understand how these ideas connect to each other and build on one another to produce a coherent whole". For example, it is possible to realize the connection between mathematical relations and matrices by expressing the relations in the form of graphs.

The "Instructional programs in school should enable all students to create and use representations to organize, record, and communicate mathematical ideas and use representations to model and interpret physical, social, and mathematical phenomena".

Graph theory is a rich source in problem-solving. We can taste so many different strategies. It is possible to make the students to think creatively. The student also, will solve the problems by graph modeling and show them graphically. Thus it is clear that including the Graphs in all levels of school Mathematics is necessary and relevant for teaching mathematical processes.

The Article 1, the first part of two-parts article by Prof. Ambat Vijayakumar, is expected to shed light on how graph theory got developed into an exciting branch of mathematics starting from the solution of a puzzle related to the land areas and bridges to the real world networks such as the brain networks, the epidemiological networks etc.. The second part of the whole article is expected to be included in a subsequent issue of the TMC Bulletin.

In the second article, Prof. Athanase Papadopoulos highlights the titanic work that Sullivan, recipient of the 2022 prestigious Abel prize awarded by the Norwegian Academy of Science and Letters, accomplished in a period of 60 years, to the exceptional influence he had on shaping 20^{th}-21^{th} century mathematics, and to all the good he has done for the mathematical community. This is also included as one of the news item in the article 4.

The announcement of the creation of Gopal Prasad Professorship by the Institute of Advanced studies (IAS), Princeton, USA, constitutes the article 3. IAS is one of the leading independent centers for theoretical research and intellectual inquiry since its establishment in 1930.

In the Article 4, Dr. D. V. Shah gives an account of the important events which occurred in the Mathematics world during last three months, which include, major breakthroughs, announcement of awards/prizes and tributes to renowned mathematicians who passed away during this period.

In the Problem Corner, Dr. Udayan Prajapati presents a solution, given by one of the readers, to the problem posed in January 2022 issue and poses a problem on Diophantine equation, for our readers. Dr. Ramesh Kasilingam gives a calendar of Academic events, planned during August, 2022 to October, 2022. Prof. S. A. Katre gives an account of TMC activities and Prof. Sudhir Ghorpade gives update on TMC- Distinguish Lecture Series.

We are very happy to bring out this first issue of Volume 4 in July 2022. We thank all the authors, all the editors, our designers Mrs. Prajkta Holkar and Dr. R. D. Holkar and all those who have directly or indirectly helped us in bringing out this issue on time.

Chief Editor, TMC bulletin.

1. Graph Theory: 1736-1936 and Beyond - Part 1

Ambat Vijayakumar
Emeritus Professor, Department of Mathematics, Cochin University of Science and Technology
Cochin - 682 022, India.
Email: vambat@gmail.com

In May 1991, the then Department of Mathematics and Statistics, Cochin University of Science and Technology, Cochin organized a 'National Symposium on Graph theory and Combinatorics' coordinated by me. Prof. S. B. Rao was one of the lead speakers. He had reached Cochin a couple of days before the seminar dates to visit important tourist places near Cochin-the Queen of the Arabian sea. Seeing our academic enthusiasm, he cut short the tour and gave a series of lectures on some open problems in self-complementary graphs. That was the beginning of a research group in graph theory in CUSAT. In 1998 we organized the first International Conference on Graph Theory, the 'Conference on Graph Connections' inaugurated by Robin J. Wilson. He is a very exciting and inspiring speaker on many aspects of mathematics and its history, author of several books including very popular ones like 'Stamping Through Mathematics' (Springer, 2001). He has also co-edited with Norman L. Biggs and E. Keith Lloyd, the book, 'Graph Theory 1736-1936' (Oxford, 1976), the title of which forms a part of the title of this article also. The year 1736 could be considered the origin of graph theory and in 1936 the first book on graph theory titled 'Theorie der endlichen und unendlichen graphen' (Theory of finite and infinite graphs) by Denés König was published.

This two-parts article is expected to shed light on how graph theory got developed into an exciting branch of mathematics starting from the solution of a puzzle related to the land areas and bridges to the real world networks such as the brain networks, the epidemiological networks etc..

Biggs [3] says "The origins of graph theory are humble, even frivolous. The problems which led to the development of graph theory were often little more than puzzles, designed to test the ingenuity rather than the stimulate the imagination. But despite the apparent triviality of such puzzles, they captured the interest of mathematicians, with the result that graph theory has become a subject, rich in theoretical results of a surprising variety and depth".

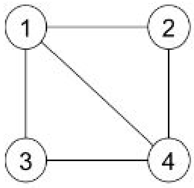
Despite such an origin, graph theory has spread its wings to many domains, both theoretically and algorithmically. It is quite exciting to know that, a team of scientists led by Martin Nowak [9], a mathematical biologist of Harvard University, currently the Director of the Program for Evolutionary Dynamics, is now studying how topology affects the evolution of population and the theory of natural selection propounded by the great Charles Darwin, during 1850s. This has resulted in the emergence of a new branch of study called 'Evolutionary graph theory' based on probability theory, graph theory and mathematical biology.

1.1 Relations, Graphs and Matrices

Relations are abundant in nature. Wherever there is a relation, there is a graph and vice versa. Given a relation, let us see how we get a graph. According to the Aristotlean logic, any two objects a, b are either related or not. We represent the objects by dots (vertices, points) and draw a line segment (edge, arc) joining a and b if they are related. Also, let us give a value 1 if a is related to b, and the value 0, if not. Hence, if there are 'n' objects and 'm' relations between them, we have a graph with n vertices and m edges and correspondingly an $n \times n$ symmetric matrix with $2m$ 1's. Thus, a graph of order n is represented by a binary, symmetric, square matrix of order n called its Adjacency Matrix.

If two graphs are isomorphic, the corresponding matrices will be similar. Traditionally, a graph G is denoted by $G = (V, E)$ where V is a non empty set and E is a collection of 2-element subsets of V (in fact those pairs of elements of V which are related). So, if $|V| = n$, and $|E| = m$, then $0 \leq m \leq n(n-1)/2$. The numbers n, m are called the order and the size of G respectively. A

Dedicated to Prof. S. B. Rao, formerly of ISI, Kolkata, and Prof. Robin J. Wilson, Emeritus Professor, The Open University, UK.



$1R2$
$1R3$
$1R4$
$2R4$
$3R4$

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Figure 1: Graph of $K_4 - e$, symmetric relation R, adjacency matrix

graph of order n with $m = n(n-1)/2$ is called a complete graph, traditionally denoted by K_n, K standing for the Polish mathematician Kazimierz Kuratowski.

Let us first consider some questions.

- A person invites six friends for a birth day party. When they enter the hall, they shake hands with each other. How many hand shakes were made?
- In a meeting, the participants decide to sit around a table, in such a way that each day they get new neighbors (persons sitting on either side of a person are called his neighbors). For how many days such a seating is possible?
- We have some chemicals to be kept in different compartments of a refrigerator so that those chemicals that may react are kept in different compartments. What will be the minimum number of compartments needed?
- We decide to color the different states of India such that states which share a common boundary are differently colored. What is the minimum number of colors required? In this case, we all know that Kerala, Tamil Nadu and Karnataka are such that any two of them have a common boundary. So, we require three colors for these three states as per the stipulations. Can we extend this three coloring to the entire country?

  Such questions can be easily solved using some innovative techniques in graph theory that we plan to discuss.

1.2 The Frivolous origin of Graph Theory - Euler and the Bridge Problem

Leonhard Euler

The Swiss mathematician Leonhard Euler (1707-1783) popularly known as the 'Analysis Incarnate', was undoubtedly a great genius who has contributed to many different, seemingly unrelated fields such as the theory of music, ship designs, complex analysis, number theory and so on. On 26 August 1735, he presented a paper titled *'Solutio problematis ad geometriam situs pertinentis'* (On the solution to a problem in the geometry of position) at the St. Peters Academy. It was actually a simple looking puzzle set in the old Prussian city of Königsberg (called Kaliningrad (Russia) since 1946) and set on both the sides of Pregel river, the islands Kneiphof and Lomse. These two islands and the two main land areas were connected to each other by seven bridges. The problem which later came to be known as the celebrated 'Königsberg Bridge Problem'(KBP) was to find a route to travel from any of the four land areas (Fig.2), travel each of the seven bridges exactly once and return to the same land area. Euler used an ingenious argument to show that such a route is not possible. In this way the year 1736 became the birth year of graph theory. In graph theoretic terminologies, the graph of the Königsberg Bridge Problem is a multi graph of order 4 and size 7 which is not Eulerian.

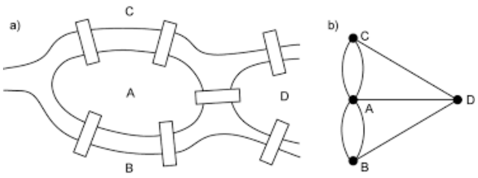
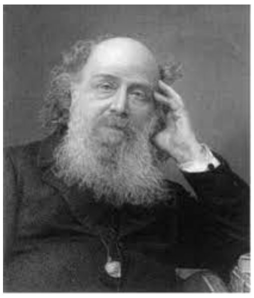


Figure 2: The graph model of KBP

It is not Eulerian because, by Euler-Hierholzer theorem, a connected graph is Eulerian if and only if every vertex is of even degree, and in the graph of the KBP, every vertex is of odd degree.

But it is a less known fact that Euler in his paper never used the term 'Graph'. It was the British mathematician J. J. Sylvester (1814-1897) who used the term 'Graph' for the first time in his paper titled Chemistry and Algebra, which appeared in Nature, in 1878.



J. J. Sylvester

It is not surprising that Euler considered this problem trivial, which he mentioned in 1736 in a letter to Carl Leonhard Gottlieb Ehler, the Mayor of Danzig, who asked him for a solution to the problem. "Thus you see, most noble Sir, how this type of solution bears little relationship to mathematics, and I do not understand why you expect a mathematician to produce it, rather than anyone else, for the solution is based on reason alone, and its discovery does not depend on any mathematical principle. Because of this, I do not know why even questions which bear so little relationship to mathematics are solved more quickly by mathematicians than by others".

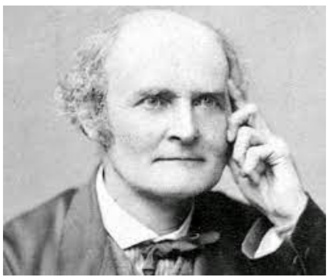
Even though Euler found the problem trivial, he was still intrigued by it. In a letter written the same year to Giovanni Marinoni, an Italian mathematician and engineer, Euler said, "This question is so banal, but seemed to me worthy of attention in that neither geometry, nor algebra, nor even the art of counting was sufficient to solve it".

Euler believed this problem was related to a topic that Gottfried Wilhelm Leibniz (1646-1716) had referred to as 'geometria situs', or 'geometry of position'. This so-called geometry of position is what is now called graph theory, which Euler introduced and utilized while solving this famous problem.

1.3 The Four Colour Problem

One of the most exciting developments in graph theory is centred around a class room problem. On 23 October 1852, the celebrated British mathematician and a Professor of Mathematics at the University College, London, Augustus De Morgan (1806 - 1871), well known for the De Morgan's laws in set theory and many other major contributions, mentioned in a letter the 'Four Colour Problem' to his teacher Sir William Rowan Hamilton (1805-1865). Incidentally, it may be of interest to note here that De Morgan was born in Madurai, Tamilnadu.

The letter starts mentioning a doubt expressed by one of his students Frederick Guthrie (for his brother Francis Guthrie) while colouring the map of England, whether four colours are sufficient to colour different countries in any map, if neighbouring countries are coloured differently. This was the origin of the much celebrated 'Four-Colour Problem' (Four Colour Conjecture - 4CC). See [16] for an interesting reading. **"4 CC: Four colours are sufficient to colour the different regions of any map drawn in the plane or on a sphere so that no two regions with a common boundary line are coloured with the same colour."**



Arthur Cayley

In 1878, another celebrated British mathematician Arthur Cayley(1821-1895) publicised this problem in a meeting of the London Mathematical Society. A proof was given by the London based barrister Alfred Kempe (1849-1922) in 1879 which remained as a 'proof' for about eleven years. In 1890, a student at Oxford University, Percy Heawood (1861-1955) found an error in Kempe's argument and proved, essentially using the 'Kempe chain' argument, the **Five Colour Theorem: Every map can be coloured with five colours.** However, reducing the number from five to four took more than eighty years. Many attempts to solve the four colour problem through various, different means, spanning about 120 years enriched not only the field of graph theory and in particular the theory of graph colouring, but the whole of mathematics.

It turned out after much research that if at all the 4CC could be proved, one needs to find an 'unavoidable set of reducible configurations'. By 1970, H. Heesch developed a very innovative method of discharging to construct the reducible configurations. Kenneth Appel (1932-2013) and Wolfgang Haken (1928) of University of Illinois, USA spent several years to develop computer programs to find the unavoidable sets of configurations. Finally they along with John Koch, a programmer, were successful in constructing about two thousand reducible configurations, which in particular involved about one thousand two hundred hours of computer time. Finally, they announced the completion of their proof of 4CC in a series of papers that appeared in 1977 [2].

4CC is one of the rare conjectures in mathematics to have received so much attention and met with so many unsuccessful proofs. One interesting episode is worth mentioning here. Thomas L. Saaty, a distinguished Professor at the University of Pittsburgh was selected for the Lester R. Ford award (known as Paul R. Halmos- Lester R. Ford award since 2012) given annually by the Mathematical Association of America for his expository paper entitled 'Thirteen colourful variations on Guthrie's Four color conjecture' which was published in the January 1972 issue of the American Mathematical Monthly. Several equivalent forms and different possible approaches to solve the 4CC were discussed in detail [11] and it was concluded that it is most unlikely that a solution to 4CC would be possible in the near future. But, within just four years 4CC became 4CT.

Paul Hoffmann [8] in his biography of Paul Erdös says that, "when the four-colour theorem was proved, Erdös (who will be introduced later) entered his calculus class with the fuel of excitement carrying two bottles of champagne. He wanted to celebrate the moment because it was a long-running unsolved problem. Then the next day, when he came to know that the proof had been done by computers, he became depressed. The reason is that now they know the theorem is true but not justified by pure mathematical reasoning".

Many other mathematicians were also worried that it might change the very nature of mathematics. Unlike the statement of the problem, the solution was found to be quite complicated. That was the first-ever theorem in history of mathematics that was proven by computers. Some mathematicians showed disagreement with the computer's logic and claimed that it would be infeasible to counter-check that huge work by hand. A simpler and efficient proof using the same ideas and still relying on computers was published in 1997 by Neil Robertson, Daniel P. Sanders, Paul Seymour, and Robin Thomas [10].

It is interesting to read the following Editorial of New York Times, dated 26 September 1976 (not the first of this kind as we see later).

> "Now the four-colour conjecture has been proved by two University of Illinois mathematicians, Kenneth Appel and Wolfgang Haken. They had an invaluable tool that earlier mathematicians lacked modern computers. Their present proof rests in part on 1,200 hours of computer calculation during which about ten billion logical decisions had to be made. The proof of the four-colour conjecture is unlikely to be of applied significance. Nevertheless, what has been accomplished is a major intellectual feat. It gives us an important new insight into the nature of two-dimensional space and of the ways in which such space can be broken into discrete portions."

It is quite exciting and surprising that seemingly much more difficult problems in graph colouring related to 4CC, including the **Heawood Map-Colouring Conjecture** proposed in 1890 for surfaces other than the sphere were settled much earlier and more easily. Gerhard Ringel (1919-2008), a German mathematician, and an American mathematician J. W. T. Youngs (1910-1970) proved this conjecture in 1968.

Ringel - Youngs theorem: The minimum number of colors necessary to color all graphs drawn on an orientable or a non orientable surface is $\gamma(\chi) = \left\lfloor \frac{7+\sqrt{49-24\chi}}{2} \right\rfloor$ where χ is the Euler characteristic.

Euler characteristic is a topological invariant, a number that describes a topological space's shape or structure regardless of the way it is bent. This relation holds for all surfaces except the Klein Bottle, which is a non-orientable surface of Euler Characteristic 0 and has minimum number of colours as 6 instead of 7 predicted by the formula.

Attempts to prove or disprove 4CC was a point of discussion among the mathematical community and even the general public. In this connection, there is an interesting episode related to Ringel. One day, a police man stopped Ringel for over-speeding. However, he identified Ringel and told him that this time you are exempted from the payment of penalty. Ringel was quite surprised at the unusual behaviour of the cop and asked him the reason. He explained that he had heard about the four colour problem and Ringel's contributions through his son who was Ringel's student!

1.4 Chess Board Problems

There are several puzzles related to the chess boards [15] that have caught the attention of mathematicians, some of which are still unsolved. Earliest such problem involving the movement of knights dates back to Guarini in 1512. A typical and well-known problem was, what is the minimum number of queens that can be placed on an $n \times n$ chess board 'dominating' the board, i.e. such that each square is either occupied by a queen or attacked by a queen? For the standard 8×8 chess board, it is known that this number is 5. But it is quite surprising that there are 4860 different ways that these five queens can be arranged to dominate the board. The Norwegian mathematician Øystein Ore (1899-1968) converted these problems to graph theoretic ones and initiated the study of 'Domination theory in graphs'.

The knight's tour problem is the problem of finding a knight's tour - a sequence of moves of a knight on a chessboard such that the knight visits every square exactly once. Which $n \times n$ chess board admits a knight's tour? is an interesting and easy combinatorial problem.

The problem of finding a cycle in a graph which passes through all its vertices is referred to as the 'Hamiltonian Problem', named after the legendary Irish mathematician William Rowan Hamilton (1805-1865). This problem is one of the classical problem which is NP-complete. In 1857, Hamilton invented the 'Icosian game'- to find such a cycle along the edges of a dodecahedron. He had to commercialise this game for his earnings.

1.5 Chemical Graph Theory

A connected graph without cycles is called a tree. Decision tree, genetic tree, sorting tree etc are some examples of trees from different fields. Arthur Cayley found that the problem of counting different isomers of saturated hydro carbons is same as counting the number of n- labelled trees. Hydrocarbons are chemical compounds that consist of only carbon and hydrogen atoms. Those hydro carbons with only single bonds are called alkanes. Methane, Ethane, Propane etc. are some examples and they have the general chemical formulae C_nH_{2n+2}. Since its planar representation is a connected graph with $3n+2$ vertices and $3n+1$ edges, it is a tree.

Cayley's Tree formula (1889): The number of labelled trees on n vertices is n^{n-2}.
This formula equivalently counts the number of spanning trees (spanning subgraphs which are trees) of a complete labelled graph on n vertices. Arthur Cayley was probably the first to publish results that consider molecular graphs, as early as in 1874, even before the introduction of the term 'graph'. For the purposes of enumeration of isomers, Cayley considered 'diagrams' made of points labelled by atoms and connected by links into an assemblage. Such earlier studies have now ushered a new branch called Chemical Graph Theory [14].

1.6 Spectral Graph Theory - Molecular descriptors and Topological Indices

In [13], the authors have explained in detail the significance of the molecular descriptors. Essentially, the molecules are treated as real bodies and chemical information involved is extracted mathematically and represented as numbers. There are plenty of software for the calculation of these descriptors. One such descriptor was derived from the molecular orbital theory initiated by Erich Huckel in 1930s. Basically it was to determine energies of molecular orbitals of Pi electrons of Pi delocalised molecules such as ethylene, benzene etc. This idea was extended to 'Graph Energy' [6] by a Serbian mathematical chemist Iván Gutman and his team in late 1980s by first considering molecular graphs.

Eigenvalues of the adjacency matrix of a graph - spectra of a graph, which are real numbers, the adjacency matrix being symmetric, play a significant role here. It was found to be a good molecular descriptor, and so too, the sum of the absolute values of the eigenvalues, called the graph energy. The spectra and the energy have turned out to be an exciting branch of research. The classical question, can you hear the shape of the drum, could be asked here also in the sense that, which graphs are determined uniquely by their spectra? This remains a very challenging question. Integral graph - a graph whose spectrum consists only of integers, heredity properties of graph energy, distance energy etc. are some promising areas of research.

The spectra of the Laplacian matrices $L = D - A$, where D is the degree matrix and A is the adjacency matrix associated with a graph, are finding applications in machine learning algorithms and the notion of algebraic connectivity - the second smallest Laplacian eigenvalue of a graph - finds applications in the study of separability of brain networks in resting-state functional MRI data from Attention-Deficit/Hyperactivity Disorder patients [5].

There is another type of molecular descriptor, called the 'Topological index'. This is calculated from the molecular graph of the compounds and serves as a graph invariant - a graph property which is invariant under isomorphism. Topological indices are used in the development of quantitative structure-activity relationships (QSARs) in which the biological activity or other properties of molecules are correlated with their chemical structure.

The 'Wiener index' named after an American chemist Harry Wiener is the oldest topological index, introduced in 1947. It is defined as the sum of the lengths of the shortest paths between all pairs of vertices in the graph, the notion developed initially for a chemical graph has now been adopted for any connected graph. This index is closely related to the boiling points of alkanes. An interesting question posed and answered by Gutman and Yeh [7] was to determine, which numbers can be the Wiener index of graphs? It is proved that the numbers 2 and 5 are the only exceptions.

Another topological index which is well studied is the 'Hosoya index', named after the Japanese chemist Haruo Hosoya, introduced in 1971. Hosoya index (Z index) of a graph is the number of matchings in it. A k- matching in a graph is a collection of k non-adjacent edges. As an example, Hosoya index of K_4 - the complete graph of order 4 - is ten and the maximum value of Hosoya index is attained, among all graphs of a given order, for complete graphs. The notion of 'matching polynomials' and many other graph polynomials such as independence polynomials, chromatic polynomials etc. are also of significance in Graph theory.

1.7 Fullerene Graphs - Buckminster Fullerene

Carbon has existed in nature much before humans. It was believed till recently that carbon existed in nature in only two different forms, the diamond and the graphite, with totally different properties. But another class of carbons, called 'fullerenes' were discovered around early 1980s. These molecules were denoted by their empirical formulae Cn, where n corresponds to the number of carbon atoms. Buckminster fullerene C60- molecule of the year 1991- is the most well studied member of this family and named after the American architect Buckminster. It looks like a soccer ball and its huge structure can be seen in the Montreal city of Canada. Harold Kroto, Robert Curl and Richard Smalley of USA were awarded the Nobel prize in chemistry in 1996 for its discovery.

Figure 3: A model of 'Bucky Ball' in Montreal city, Canada

Fullerene graphs, whose vertices represent carbon atoms and edges represent bonds between atoms, are cubic, three-connected planar graphs with twenty hexagons and twelve pentagons. Being a cubic graph, it has a unique planar embedding. It can be easily proved using the Euler's theorem for a connected planar graph that Fullerene graphs exist for all $n \geq 24$ and $n = 20$.

The difference between the largest and the second largest eigenvalues of a graph is called the 'spectral gap' $s(G)$ of a graph. The largest eigenvalue of the fullerene graph is 3. Both the smallest eigenvalue and $s(G)$ play significant roles in the stability analysis of fullerenes. In 2005, Stevanovic and Caporossi proved a Graffiti conjecture that the spectral gap of the fullerenes is at most 1. One of the recent results asserts that every fullerene graph is 5-cyclically connected. The independence number, diameter, hamiltonicity, topological indices etc. of fullerene graphs are also studied.

1.8 Strongly Regular Graphs, The Euler Spoilers and The Shrikhande graph

S. S. Shrikhande

R. C. Bose

We shall now discuss some important contributions of R. C. Bose (1901-1987) and S. S. Shrikhande (1917-2020). We need first, the notion of 'Strongly Regular Graphs'. A strongly regular graph is a k-regular graph on n vertices such that any two adjacent vertices have λ common neighbours and any two non adjacent vertices have μ common neighbours, denoted by $SRG(n,k,\lambda,\mu)$.

This class of graphs were introduced by R. C. Bose in 1963 in the context of balanced incomplete block designs and partial geometries. The cycle on 5 vertices is $SRG(5,2,0,1)$. The complement of an SRG is also an SRG with different set of parameters. It can be seen that the four parameters cannot be totally independent, and applying the double way counting, the parameters in fact satisfy $k(k-\lambda-1) = (n-k-1)\mu$. Also, a graph G of order n, which is not a complete graph or a totally disconnected graph is strongly regular if and only if its adjacency matrix A satisfies the conditions,

$$AJ = JA = kJ \; ; \; A^2 + (\mu-\lambda)A + (\mu-k)I = \mu J$$

where I denotes the identity matrix and J the matrix of ones, both of order n.

A connected, regular graph is a strongly regular graph if and only if it has three distinct eigenvalues. These eigenvalues can be expressed in terms of the parameters as follows. $\lambda_1 = k$, $\lambda_2 = \frac{1}{2}\left[(\lambda-\mu) + \sqrt{(\lambda-\mu)^2 + 4(k-\mu)}\right]$ and $\lambda_3 = \frac{1}{2}\left[(\lambda-\mu) - \sqrt{(\lambda-\mu)^2 + 4(k-\mu)}\right]$.

Thus, the eigenvalues of some named graphs such as $SRG(10,3,0,1)$ - the Petersen graph, $SRG(16,6,2,2)$ - the Shrikhande graph (Fig. 4), $SRG(16,5,0,2)$ - the Clebsch graph and $SRG(50,7,0,1)$ - the Hoffman Singleton graph could be easily computed using the above formula as {3, 1, -2}, {6, 2, -2}, {5, 1, -3} and {7, 2, -3} respectively.

It is quite exciting to note that, the construction of an $SRG(99,14,1,2)$, called the Conway's 99-graph problem named after John H. Conway, who passed away on 11 April 2020 due to post covid-19 issues, remains open even now.

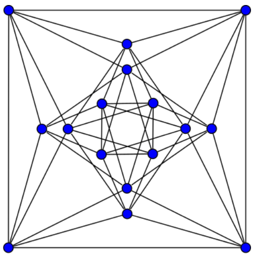


Figure 4: The Shrikhande Graph

Strongly regular graphs with $\lambda = 0$ and $\mu = 1$ and girth (length of the shortest cycle) = 5 are called 'Moore graphs', named after Edward F Moore - an American computer scientist and inventor of the Moore Finite State Machine. The only known Moore graphs are the cycle C5, the Petersen graph and the Hoffman Singleton graph. The only possible parameters for which a Moore graph could exist is (3250, 57, 0, 1). It is unknown even now if such a graph exists. See Dalfó, C. (2019), A survey on the missing Moore graph , Linear Algebra and its Applications, 569: 1-14.

The legendary mathematician Sharadchandra Shankar Shrikhande (19 October 1917 - 21 April 2020), a renowned design theorist and graph theorist, was born in Sagar, Madhya Pradesh, India. In the year 1950, he got Ph.D. from University of North Carolina, under the guidance of R. C. Bose. He had served in the Banaras Hindu University and the University of Mumbai. He was one among the three 'EULER SPOILERS' who along with R. C. Bose and E. T. Parker made a historic disproof of the celebrated Euler's conjecture on mutually orthogonal Latin squares, motivated by the '**Thirty Six Officers Problem**': Is it possible to arrange thirty six officers from six different regiments and of six different ranks in a rectangular array such that each row and each column has one officer from each regiment and one of each rank? Such an array of symbols/numbers, each symbol occurring exactly once in each row and column, which resembles a matrix is called a Latin rectangle (square). This problem can now be rephrased as, does there exist a pair of Mutually Orthogonal Latin (MOL) squares (Graeco-Latin square) of order six. Garson Tarry (1843-1913), a French mathematician proved in 1901 that such a pair of Latin squares exists and paved the way to some important developments in combinatorics.

The Euler's conjecture was a consequence of his unsuccessful attempts to solve the thirty-six officers problem and the simple observation that there are no pairs of MOLs of order two. Euler in 1782 conjectured that: **There do not exist two mutually orthogonal Latin squares of order $4n+2$ for any n.**

In the paper [4] they disproved this conjecture and proved that there do exist a pair of MOLs except for $n = 0,1$. This historic breakthrough was publicised in a newspaper as well (to my knowledge, the first of its kind).

In recognition of the significance of the then recent disproof of a famous conjecture concerning Latin squares made by Euler in 1782, an editorial in the New York Times of April 27, 1959 stated that, 'it would be a serious mistake to suppose that modern mathematics is far from real life. Actually there has never been a time in history when mathematics was so widely applied in so many different fields for so many vital purposes as is true now.'

We shall now discuss the Shrikhande's theorem, which motivated the definition of the Shrikhande Graph [12].

Let $M = \{1,2,3,\dots m\}$. Consider the graph $L_2(m)$ whose vertex set is $M \times M$ and two vertices are adjacent if the 1st coordinates or 2nd coordinates are the same. This is exactly the graph on the cells of an $m \times m$ chess board, two cells are adjacent if they are in the same row or column. We can easily see that, the resulting graph is an $SRG(m^2, 2(m-1), m-2, 2)$.

Shrikhande's Theorem : Let G be a strongly regular graph with parameters $(\mathbf{m^2}, \mathbf{2(m-1)}, \mathbf{m-2}, \mathbf{2})$. Then G is isomorphic to $\mathbf{L_2(m)}$ except when $\mathbf{m = 4}$.

This exceptional graph is the Shrikhande Graph which is an $SRG(16,6,2,2)$. Some of its nice properties are: It is Eulerian, Hamiltonian, toroidal, non-bipartite, self-centred with radius 2, has chromatic number 4, chromatic index 6 and so on. It is associated with a balanced incomplete block design. It is a Cayley graph and its automorphism group is of order 192 (proof of this is nontrivial!). However, we still don't know its chromatic polynomial - the number of different ways of colouring the vertices of a graph using t colours, which can be expressed as a polynomial in the variable t.

Is the Shrikhande graph uniquely determined by its spectra? The interesting answer to which we are led by the above theorem is, NO. It is known that the spectrum of Shrikhande graph is $\{6, 2, -2\}$, while on the other hand by the theorem there exists another strongly regular graph which is not isomorphic to the Shrikhande Graph whose spectrum is also $\{6, 2, -2\}$.

1.9 A meeting with Paul Erdös

Paul Erdös (1913-1996), the legendary Hungarian mathematician has contributed extensively to many different areas of mathematics - number theory, probability theory, extremal graph theory, set theory etc.. Described as the most prolific mathematician of the twentieth century, he published more than 1500 research papers and inspired the creation of 'Erdös Number ' which measures his 'collaborative distance' with other mathematicians through research papers. He was awarded doctorate in 1934, at the age of 21. He stunned the mathematical community at a very young age by giving a simple proof of **Chebyshev's Theorem: A prime can always be found between any integer (greater than 1) and its double.** Erdös has plenty of profound theorems to his credit, of which we shall mention one of the easiest to explain.

The **friendship theorem** which he proved along with Alfred Rényi, and Vera T. Sos in 1966, may be informally stated as follows: If a group of persons has the property that every pair of persons has exactly one friend in common, then there must be one person who is a friend to all the others. A beautiful proof is in [1], for which the authors Aigner and Ziegler, got the 'Leroy P. Steele Prize for Mathematical Exposition' of American Mathematical Society for their book, in 2018. This book is dedicated to Erdös who during a lecture in 1985, said, "You don't have to believe in God, but you should believe in The Book."

A photo of Paul Erdös in his office, taken by the author

Erdös had direct collaboration with about 509 mathematicians from different parts of the world and hence all of them had Erdös number 1. The list included, Alfred Renyi, Vera T. Sos, Ronald Graham, Janos Pach, Joel Spencer, Bela Bollobas, Peter J. Cameron, all celebrated mathematicians, and many other giants. Those with Erdös number 2, around 12600 in number, included G. H. Hardy, Albert Einstein, Manjul Bhargava and Terence Tao etc.. Through S. B. Rao, my Erdös number is also 2.

One of Erdös's rare practices was to offer cash prizes to those who solved his open problems. One such problem which is still unresolved is, **Erdös Conjecture on Arithmetic Progressions : If the sum of the reciprocals of the members of a set A of positive integers diverges, then A contains arbitrarily long arithmetic progressions.**

It is interesting to know that in 2021, a minor asteroid was formally named as 'Erdöspál' to commemorate Erdös, with the citation describing him as a Hungarian mathematician, much of whose work centered around discrete mathematics.

His following observation about Srinivasa Ramanujan is worth mentioning. "Unfortunately, I never met Ramanujan. He died when I was seven years old, but it is clear from my papers that Ramanujan's ideas had a great influence on my mathematical development." (from his article, Ramanujan and I, reprinted in Resonance, March, 1998).

One of the most exciting moments in my professional life is the meeting with Erdös in his office in the then Mathematics Research Institute (renamed as Alfred Renyi Instititute), Budapest, during my visit there in October 1994 through an exchange programme of UGC. During this visit, I could attend his talk and interact with him, Vera T. Sos, G. O. H. Katona, his son G. Y. Katona, and M Simonovits as well. One day, I gave him a picture post card depicting 'Kathakali'- a story play conducted mainly during temple festivals in Kerala, with elaborate colourful make up, costumes and face masks. He could easily identify the picture in the post card as a 'soft' character like Arjuna or Krishna because of the green colour paint in the face mask. When I told him that I am from Kerala, he asked me whether I knew that Kerala is internationally famous for three things - Communism, Christianity and Public literacy. I was not competent enough then (even now) to appreciate the works of Erdös, but had a childish desire to get his autograph. So, I requested him for his reprints. He took me to his room and asked me to take whatever I needed from the hundreds of reprints lying on the floor! I took a couple of them and on a piece of paper got his signature also. I never thought that I would ever refer his papers. But, later after ten years, one of my students got a tip from one of those papers to start her research. Such are definitely some nostalgic and exciting moments in our research life.

He once said, "I want to be giving a lecture, finishing up an important proof on the blackboard, when someone in the audience shouts out, 'What about the general case?'. I'll turn to the audience and smile, 'I'll leave that to the next generation,' and then I'll keel over".

That was exactly what happened on 20 September 1996, at the age of 83; he had a heart attack and died while attending a conference in Warsaw.

He had promised me that he would visit Cochin, but it was not to happen.

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TMC DISTINGUISHED LECTURE SERIES IN ITS SECOND YEAR

Sudhir Ghorpade, Department of Mathematics, IIT Bombay, Mumbai.
Email: sudhirghorpade@gmail.com

Already in its second year, the TMC Distinguished Lecture Series (in short, TMC DLS) began in the calendar year 2022 with a talk delivered jointly by **Prof. Samit Dasgupta** (Duke University, USA) and **Prof. Mahesh Kakade** (Indian Institute of Science, Bangalore) entitled *On Brumer-Stark units and Hilbert's 12^{th} problem.* This was followed with talks by **Prof. Neena Gupta** (Indian Statistical Institute, Kolkata) on $\mathbb{G}_a$*-actions and their applications*, **Prof. Melanie Matchett Wood** (Harvard University, USA) on *Finite quotients of 3-manifold groups*, **Prof. Rekha Thomas** (Washington University, Seattle, USA) on *Graphical Designs*, and Prof. Amie Wilkinson on *Asymmetry in dynamics.*

The TMC DLS had its first lecture in October 2020 and since then it has featured virtual colloquiums by some of the best researchers and expositors around the world. These are generally held once a month. The last four issues of the Bulletin contained brief write-ups on the lectures in this series by Professors Yves Benoist (Paris), Bernd Sturmfels (Leipzig/Berkeley), Nalini Anantharaman (Strasbourg), Alex Lubotzky (Jerusalem), Scott Sheffield (MIT), Karen Smith (Michigan), Daniel Wise (Montreal), Mikhail Lyubich (Stoney Brook), Tadeshi Tokieda (Stanford), Siddharth Mishra (Zürich), Noga Alon (Tel Aviv/Princeton), Mladen Bestvina (Utah), Laura De Marco (Harvard), and Irit Dinur (Weizman). The typical format of most of these talks is to first release a pre-recorded video of the lecture, solicit questions from registered participants, and then hold a live interaction with the speaker about two weeks after the release of the video lecture. In the future, it is proposed to also include live lectures at selected Institutes across the country, especially by mathematicians based in India, and subsequently, release a recording of the lecture.

The TMC DLS is organized by The (Indian) Mathematics Consortium, and it is co-hosted by IIT Bombay and ICTS-TIFR Bengaluru. More information about the TMC DLS is available at: *https://sites.google.com/view/distinguishedlectureseries/*

The videos of the talks held thus far are available on the TMC YouTube Channel at: *https://www.youtube.com/channel/UCoarOpo_-9fgzFWDap6dFFw/*

2. Dennis Sullivan: Mathematics and physics; manifold and space

Athanase Papadopoulos
Institute de Recherche Mathématique, Univ. of Strasbourg, CNRS, Strasbourg Cedex, France
Email: papadop@math.unistra.fr

We do not have a Nobel prize in mathematics, but we have two distinctions which are at least comparable to it, the Fields medal and the Abel prize. Unlike the Fields medal, the Abel prize has no age limitation; it is usually awarded for long-term achievements.

The history of the Abel prize is interesting. Sophus Lie, who, like Niels Henrik Abel, was Norwegian, proposed the establishment of this prize already in 1899, after he learned that the Nobel Prizes (which were to be awarded for the first time in 1901 by the Swedish and the Norwegian Academies) will not include mathematics. The first Abel prize was planned to be part of the events celebrating the 100th anniversary of Abel's birth (August 5, 1802). The project was then delayed, and eventually aborted for political reasons, in particular the dissolution of the union between the kingdoms of Norway and Sweden (1905). The idea of this prize arose again at the beginning of the second millenium, and in August 2001, the Norwegian government announced that the prize would be awarded starting in 2002, again in relation with Abel, this time for his two-hundredth anniversary. Instituting the prize was slightly delayed, and it was awarded for the first time in 2003, to Jean-Pierre Serre, who had been recipient of the Fields medal about 49 years before (at age 27). It has been given every year since then, and the names of the recipients include Michael Atiyah and Isadore Singer (joint prize), S. R. Srinivasa Varadhan, Misha Gromov, John Milnor, Yakov Sinai, John Nash and Louis Nirenberg (joint), and Andrew Wiles, for their outstanding contributions in various fields of mathematics. The name of Dennis Sullivan was like a gap in this list, and it was filled in March 2022.

It is impossible to do justice in a few pages to the titanic work that Sullivan accomplished in a period of 60 years, to the exceptional influence he had on shaping 20th-21th century mathematics, and to all the good he has done for the mathematical community. In this report, after a brief *Vita*, I have chosen to start by reviewing the three lectures that Sullivan gave at three International Congresses of Mathematicians (Nice 1970, Vancouver 1974 and Berkeley 1986). These lectures reflect some of his subjects of interest during the first part of his career. Then, I will present a list of the incredible amount of topics on which he worked, with special emphasis on dynamics and fluid mechanics. From time to time I will quote some emails I have exchanged with him over the years. Going through this correspondence, I see in hindsight that he was always answering my questions, mathematical or not, and his remarks were always compelling. I will mention in particular some thoughts he shared recently with me and with several of his colleagues and friends on his view on what is important in mathematics.

Born in 1941, in Port Huron (Michigan), Sullivan was brought up in Houston (Texas). He entered Rice University, first as a student of chemical engineering. It was the discovery of topology that made him change his mind and shift to mathematics. In an interview with Shubashree Desikan which appeared in *The Hindu* on March 24, 2022, Sullivan, speaking informally about this episode, says: "At Rice University, all the science students, electrical engineers and all the others, took math, physics and chemistry. In the second year, when we did complex variables, one day, the professor drew a picture of a kidney-shaped swimming pool, and a round swimming pool. And he said, you could deform this kidney-shaped swimming pool into the round one. At each point, the distortion is by scaling. A little triangle at this point goes to a similar triangle at the other point. We had a formula for the mapping, because we were taking calculus, and we had a notation for discussing it. This was like a geometric picture. This mapping was essentially unique. The nature of this statement was totally different from any math statement I had ever seen before. It was, like, general, deep, and wow! And true! So then, within a few weeks, I changed my major to math."

Sullivan obtained his PhD at Princeton University in 1966, with a dissertation titled *Triangulating homotopy equivalences*, with William Browder as advisor. In the few preceding years, algebraic topology had been invigorated with the introduction of new techniques and with a series of outstanding results on the classification of manifolds. Let me say a few words on this period.

The notions of fiber space and fiber bundle became central around the year 1950. Soon after, Serre introduced in his thesis (1951) the crucial idea of using spectral sequences to study the homology of fiber spaces. In 1952, Mikhail M. Postnikov obtained his famous reconstruction of the cohomology of a space from its homotopy invariants. René Thom, in 1954, obtained a classification of manifolds up to cobordism. In 1956, Milnor proved the existence of exotic differentiable structures on the 7-dimensional sphere. The higher-dimensional analogue of the Jordan curve theorem (the so-called Jordan–Schoenflies theorem) was obtained in 1959-1960 (works of Barry Mazur, Marston Morse and Morton Brown). In 1960, Steven Smale proved the Poincaré conjecture (PL and smooth categories) for dimensions ≥ 7 (and then ≥ 5). In 1961, Christopher Zeeman obtained a proof of the same conjecture for dimension 5, and in 1962 Stallings gave a proof for dimension 6. In 1961, Milnor disproved the so-called *Hauptvermutung der kombinatorischen Topologie* ("main conjecture of combinatorial topology"), a conjecture formulated in 1908 by Ernst Steinitz and Heinrich Franz Friedrich Tietze, asking whether any two triangulations of homeomorphic spaces are isomorphic after subdivision. In 1965 Sergei P. Novikov proved that the Pontryagin classes with rational coefficients of vector bundles are topological invariants.

Sullivan started working on topology amid all this booming activity. In his thesis, he gave an obstruction to deforming a homotopy equivalence of piecewise linear manifolds to a PL homeomorphism, obtaining further cases where the Hauptvermutung is true. This result is called "the characteristic variety theorem." It was used to provide numerical invariants that classified the combinatorial manifolds in a homotopic type. He summarized his results on this topic in his short paper *On the Hauptvermutung for manifolds* [2] which appeared in 1967 and for which he was awarded the Oswald Veblen prize of the AMS.

After his PhD, Sullivan worked successively at the University of Warwick (1966-1967), the University of California at Berkeley (1967-1969) and MIT (1969-1973). In the year 1973-74, he was invited as a visiting professor at the Université de Paris-Sud, Orsay. The next year, he became a permanent member of the Institut des Hautes Études Scientifiques at Bures-sur-Yvette. The institute is a couple of kilometers from the Orsay campus. Remembering that period, I find in an email from Sullivan (2015): "Grothendieck left IHÉS around 1970. Quillen visited IHÉS from MIT during the year 1972-1973. I visited IHÉS and Orsay from MIT during the year 1973-1974. It was a splendid place to do Math. IHÉS offered Grothendieck's vacated position to Quillen who declined. IHÉS offered it to me and I grabbed it."

In a 2019 email, Sullivan writes: "My first math hero, as a grad student in Princeton, was René Thom. When IHÉS offered me a professorship, I was honored to accept it and to become Thom's 'colleague'." He adds: "My second math hero starting in December 71 was the Mozart-like figure Bill Thurston. During the next decade I was in France and Bill was in the US but we had a quite fruitful and intense interaction." I shall say more about Sullivan's relation with Thurston below.

In 1981, Sullivan was appointed to the Einstein Chair at the Graduate Center of the City University of New York. He kept his position at IHÉS on a part-time basis and started spending half the year in France and the other half in the US, until 1996, where he took up a professorship at the State University of New York at Stony Brook, again on a part-time basis, keeping his Einstein Chair at CUNY, where he conducts, since his appointment there, a weekly seminar on geometry in the broadest sense whose sessions are known to last for several hours (sometimes all day). For me, and I assume it is the same for many others who know this place, the Graduate Center of CUNY is the central point where the heart of New York is beating.

To talk about Sullivan's work, I start with the lecture that he gave at the 1970 ICM, which took place in Nice. The invitation to that Congress came just after Sullivan proved (concurrently with and independently of D. Quillen) the Adams conjecture, which concerns the homotopy theory of sphere bundles associated with vector bundles. The conjecture was considered as one of the most important conjectures in topology. Whereas this led Quillen to develop algebraic K-theory, Sullivan's approach was based on the "arithmetization" of geometry and topology, more precisely on the introduction of Galois theory in the geometry of manifolds in the form of concepts like localisation, rationalization, profinite and p-adic homotopy theory. Sullivan's proof of Adam's conjecture, which he obtained in 1967, is based on the construction of a functor from abstract algebraic varieties into profinite homotopy theory. This approach led him to the study of the absolute Galois group itself through its actions on new geometric objects, and at the same time gave rise to strong relations between number theory and homotopy theory. In fact, Sullivan studied the action of the absolute Galois group in homotopy theory via classifying spaces and Postnikov towers. The ICM talk is titled *Galois symmetry in manifold theory at the primes* [4]. A more detailed version of this work appeared in the paper [5], in which Sullivan describes his (still open) unrequited *Jugendtraum* (childhood dream). Let me quote from the introduction to that paper some sentences which are characteristic of Sullivan's personal thinking, based on rich and inspiring analogies: "We are studying the structure of homotopy types to deepen our understanding of more complicated or richer mathematical objects such as manifolds or algebraic varieties. The relationship between these two types of objects is I think rather strikingly analogous to the relationship in biology between the genetic structure of living substances and the visible structure of completed organisms or individuals. The specifications of the genetic structure of an organism and of the homotopy structure of a manifold have similar texture; they are both discrete, combinatorial, rigid, interlocking and sequential."

Sullivan's second ICM talk (Vancouver 1974) is titled : "Inside and outside manifolds" [6]. The title is characteristic of Sullivan's taste for literary turns of phrase. The paper has two parts, which express his two approaches at the time for the exploration of manifolds: topology and dynamical systems. The first part, on "outside" manifolds, is an exposition of the complete classification theory of simply-connected manifolds of dimension at least 5. The main detailed reference for this part is a set of notes on geometric topology based on lectures that Sullivan gave at MIT in 1970, carrying the subtitle "Localization, periodicity, and Galois symmetry" [3]. These notes were widely circulated and they had a major influence on algebraic and geometric topology, not only in the West but also in the Soviet Union where they were translated into Russian just after Sullivan's ICM 1974 talk. In the second part of the lecture, "Inside manifolds", Sullivan presents a series of qualitative results on dynamics (in particular Smale's Axiom A, structural stability, and the asymptotic properties of leaves of foliations) within individual manifolds. This second part, as Sullivan writes, "focuses attention on the classical goals and problems of *analysis situs*."

A new version of the 1970 MIT notes was edited in 2005 by Andrew Ranicki [3]. It concludes with a 10 pages postscript by Sullivan in which he recounts, in his personal literary style, the genesis and later developments of these notes, as well as episodes from his own mathematical education and his interaction with the mathematicians who have inspired him. René Thom, who, as we recalled, was Sullivan's first hero, is abundantly quoted. The postscript contains several open problems and conjectures. At several points, the tone is philosophical. In his book *Surgery on compact manifolds*, C. T. C. Wall writes about these notes: "It is difficult to summarise Sullivan's work so briefly: the full philosophical exposition should be read." The postscript is punctuated with details on Sullivan's family life.

After recalling his work in the 1970s, Sullivan writes in these notes: "About this time dynamical systems, hyperbolic geometry, Kleinian groups, and quasiconformal analysis which concerned more the geometry of the manifold than its algebra began to distract me (see ICM report 1974), and some of the work mentioned above was left incomplete and unpublished." This mention of quasiconformal mappings brings us to his 1986 ICM talk titled *Quasiconformal homeomorphisms in dynamics, topology, and geometry.*

Sullivan's third ICM talk (Berkeley 1986) [8] is a survey of results and conjectures, mainly due to him, on quasiconformal homeomorphisms and their use in four different contexts. Here, a homeomorphism $\varphi : X \to Y$ between metric spaces $(X, | \, |)$ and $(Y, | \, |)$ is said to be (K-)quasiconformal if for some $K > 0$

$$\limsup_{r \to 0} \frac{\sup |\varphi(x) - \varphi(y)| \text{ where } |x - y| = r \text{ and } x \text{ is fixed}}{\inf |\varphi(x) - \varphi(y)| \text{ where } |x - y| = r \text{ and } x \text{ is fixed}} < K.$$

The four contexts in which quasiconformal mappings are studied in this paper are the following:
(1) Feigenbaum's numerical discoveries in 1-dimensional dynamics. Here, quasiconformal homeomorphisms are used to define a distance between real analytic dynamical systems, by first complexifying them. This distance is contracting under the Feigenbaum renormalization operator.
(2) The theory of quasiconformal manifolds, including developments of de Rham cohomology, Atiyah–Singer index theory and Yang–Mills connections on these manifolds. This is joint work between Sullivan and Donaldson. Combined with other works of Donaldson and Freedman, this theory provides a complete picture of the structure of quasiconformal manifolds, namely, each topological manifold has an essentially unique quasiconformal structure, except in dimension 4 where, by results of Freedman and Donaldson–Sullivan, the statement is false.
(3) The quasiconformal deformation theory of expanding systems $f : U_1 \to U$ where U_1 is a domain of the Riemann sphere and f a d-sheeted ($d > 1$) onto covering. Sullivan writes: "The analytic classification of expanding systems of a given topological dynamics type on the invariant set is a kind of Teichmüller theory." Here, an infinite-dimensional Teichmüller space of expanding analytical dynamical systems near their fractal invariant sets is embedded in the Hausdorff measure theories possible for the transformation on the fractal, and the Hausdorff measure theories of fractals are embedded in the theory of Gibbs states.
(4) The quasiconformal theory of the geodesic flow of negatively curved manifolds via the action of the fundamental group on the sphere at infinity of the universal cover. Constant curvature is given a characterization among variable negative curvature in terms of a uniform quasiconformality property of the geodesic flow.

These three ICM talks give us an idea of part of Sullivan's mathematical interests until the year 1986. I will mention some other results of him, but before that I would like to list all the fields and the topics on which he worked.

We already mentioned his work on the topology of manifolds. His results on this topic concern, among others, the classification of manifolds in several categories: smooth, PL, topological, Lipschitz, bi-Lipschitz and quasiconformal. Closely related is his work on algebraic topology: homology and homotopy theories, including rational homotopy theory, intersection homology, differential cohomology, homotopical algebra, characteristic classes, K-theory, flat bundles, minimal models, Galois symmetry and string topology. Then come the more geometrically specialized topics: foliations, minimal surfaces, geometric structures and geometry of manifolds: affine manifolds, complex manifolds, Kähler manifolds, hyperbolic geometry, Kleinian groups, universal Teichmüller theory, laminations, the solenoid and circle packings. Under the heading "dynamics", I mention 1-dimensional dynamics, renormalization, universality, holomorphic dynamics, potential theory, optimal control, measurable dynamics, and the related topics of ergodic theory, probability, Brownian motion, Diophantine approximation, chaos and fractals (in particular, self-similar structures arising in the theory of Kleinian groups, in KAM theory and in the iteration theory of holomorphic dynamics). Finally, let me also mention in a nutshell: C^* algebras, harmonic analysis, singularity theory, field theories, sigma models, and logic.

I do not know of any mathematician who has such a broad spectrum of interests, except Leonhard Euler.

I would like to say a few more words on Sullivan's French period.

As a visitor at Orsay, in 1973-74, Sullivan gave a course on the theory of rational homotopy of differential forms that he had newly developed. During the same period, he came with a new interpretation of the Godbillon–Vey invariant, a subject that was keeping busy geometers working on foliation theory which was a hot topic at that time at Orsay. The interpretation was completely new, using the notion of currents. Sullivan included this result in his paper *Cycles for the dynamical study of foliated manifolds and complex manifolds* [7], which appeared in 1976 and which contains a wealth of other results. It was during the same year that he introduced the topologists at Orsay to Thurston's theory of surface homeomorphisms; this was the major impetus to the seminar on Thurston's work which took place at Orsay in 1976-77 and which gave rise to the famous book "Travaux de Thurston sur les surfaces."

Talking about Orsay and Bures, and on a more personal level, let me recall an episode, in 1983. I was a PhD student at Orsay. François Laudenbach, who was my thesis advisor, had asked me a question related to pseudo-Anosov homeomorphisms. I came to see him one day, with a four pages manuscript, containing a proof of a theorem which answers the question. He told me: "I will show it to Sullivan." After he talked with Sullivan, he decided that this was my PhD dissertation. (For the record, I had some trouble later on, convincing people at the registrar's office that these 4 pages constitute a thesis dissertation.)

I already quoted Sullivan saying that he had two heroes, Thom and Thurston, and I would like to say a few more words on his relation with the latter.

Sullivan was always a major promoter of Thurston's ideas, and he was probably the person who best understood, from the first years, their originality and their broadness. The main themes discussed at Sullivan's seminar at IHÉS and at his New York Einstein Chair seminar included 1-dimensional dynamics and the so-called kneading theory established by Milnor and Thurston. The other topics include Kleinian groups (discrete isometry groups of hyperbolic 3-space) and holomorphic dynamics, two topics which eventually became a single topic, after Sullivan established a complete dictionary between them. Here, again, Thurston's ideas were often at the forefront, and Sullivan spent years trying to understand and to explain them. He was the first to learn from Thurston his result on the characterization of postcritically finite rational maps of the sphere, that is, rational maps whose forward orbits of critical points are eventually periodic. The proof of this theorem, like the proofs of several of Thurston's major theorems, uses a fixed point argument for an action on a Teichmüller space. The rational map in the theorem is obtained through an iterative process as a fixed point of that map on Teichmüller space. This became a major element in Sullivan's analogies between the iteration theory of rational maps and the theory of Kleinian groups. Thurston's theorem, together with Sullivan's dictionary between the theory of discrete subgroups of $PSL(2, \mathbb{C})$ and complex analytic iteration, constitute now the two most fundamental results in the theory of iterations of rational maps. Regarding this theory, in 1985, Sullivan published a paper in which he gave the proof of a longstanding question formulated by Fatou and Julia in the 1920s and which became known as the Sullivan's No-wandering-domain Theorem. This theorem says that every component of the Fatou set of a rational function is eventually periodic. A fundamental tool that was introduced by Sullivan in his proof is that of quasiconformal mappings, the main topic of his 1986 ICM lecture and one of the major concepts used in classical Teichmüller theory.

Let me also mention that in the realm of conformal geometry, Thurston introduced the subject of discrete conformal mappings, and in particular the idea of discrete Riemann mappings. In 1987, Sullivan, together with Burton Rodin, proved an important conjecture of Thurston on approximating the Riemann mapping using circle packings. This result, the authors write, is in the setting of Thurston's "provocative, constructive, geometric approach to the Riemann mapping theorem," see [1].

There is one field which I did not stress upon yet; this is physics.

Sullivan is also a physicist, in the tradition of Euler and Riemann, who were also physicists. In fact, Riemann considered himself more a physicists than a mathematician. He was thoroughly involved in gravitation, electricity, magnetism and electrodynamics and he adopted a completely physical approach to the theory of functions of a complex variable. In a note contained in his *Collected Works*, he writes: "My main work consists in a new formulation of the known natural laws–expressing them in terms of other fundamental ideas–so as to make possible the use of experimental data on the exchanges between heat, light, magnetism, and electricity." During his stay at IHÉS, Sullivan was discussing with physicists, Oscar Lanford, Henri Epstein, David Ruelle, etc. In the Postface to the 2005 edition of his 1970 MIT notes, he writes: "At a physics lecture at IHES in 1991 I learned the astonishing (to me) fact that the fundamental equations of hydrodynamics in three dimensions were not known to have the appropriate solutions."

This brings us to hydrodynamics, one of Sullivan's favorite topics, and in particular, the Navier–Stokes equation. This is the main partial differential equation which describes the flow of an incompressible fluid. It is good to remember here that a special form of this equation was discovered by Euler back in the 18th century (Euler's equation is special in that it does not take viscosity into account). In the interview that appeared in *The Hindu* which we already quoted, Sullivan, talking about the Navier–Stokes equation, declares that his fascination for this subject goes back to his youth spent in Texas. He says, again with his informal tone: "If you're in Texas, as a student of chemical engineering, there's the petrochemical industry, the oil industry, and organic chemistry and plastics, all around Houston. If you are good in science, and you work on that and become an engineer you can get a good job and have a nice work at a research center. So it's a good thing to do. In fact, during the summers, I had jobs at various such places. Once I had to study the computer methods that they were using to do what's called secondary recovery. You know, when they find oil, because of the pressure, if they drill a hole, the pressure makes it shoot up, right. But after they drill for 20 years, the pressure goes down. What they do then is go to another part of the field, and they drill and they pump in water to create pressure that will push the oil back to their wells, and for this they have to solve the linearised version of the Navier–Stokes equation. I didn't know that name then, but it's a linearised version of the Navier–Stokes equation. While at the summer job where I was studying the possible computer programs I had a certain question there. That was around 1960. [...] So in a sense, I was aware that there's this huge industry related to fluid flow through porous media. It was astonishing to me to find out as I found out in the 1990s, that those equations in three dimensions, the beautiful equations, are not solved."

In a recent email (2021), Sullivan writes on the same subject: "Back to 1991-92: After a talk at IHES by computational physicists using Fourier modes to numerically analyze the 3D incompressible Navier–Stokes Equation one learned about the lack of a mathematical answer to the question of long time existence of solutions in certain classes. That this beautiful and widely used equation that designed airplanes helped to more efficiently transport oil and to understand how to stabilize aneurisms in or near the brain or heart etc. held such mysteries was astonishing. For example, why was this 3D problem so hard? The same Navier–Stokes problem in 2D could be treated by exactly the same Calderon–Zygmund analysis tools that treated the Beltrami equation, so important in the Ahlfors–Bers treatment of Teichmüller theory. In addition half of the non-trivial theory of quasiconformal mappings in 2D extended to higher dimensions as in Mostow rigidity for 3D."

Let me also recall that the Navier–Stokes equations are among the Clay Mathematics Institute Millennium Prize problems. The statement of the Clay problem is the following: "Waves follow our boat as we meander across the lake, and turbulent air currents follow our flight in a modern jet. Mathematicians and physicists believe that an explanation for and the prediction of both the breeze and the turbulence can be found through an understanding of solutions to the Navier–Stokes equations. Although these equations were written down in the 19th Century, our understanding of them remains minimal. The challenge is to make substantial progress toward a mathematical theory which will unlock the secrets hidden in the Navier–Stokes equations."

Talking about physics, let me also mention that Sullivan's work on 1-dimensional dynamics, period doubling and chaos was motivated by physical experimentation. In the Postface to his MIT notes, he writes: "For example, one new project begun in the 1980s in fractal geometry – the Feigenbaum universal constant associated to period doubling – presents a new kind of epistemological problem. Numerical calculations showed that a certain mathematical statement of geometric rigidity in dynamics was almost certainly true, but the available mathematical technique did not seem adequate. Thus assuming the result was true, there had to be new ideas in mathematics to prove it. The project consumed the years up to the birth of my second son Thomas in July '88."

Recently (2021), Sullivan, emailed me together with a few friends and colleagues some basic and fundamental questions and thoughts on "What is a manifold", describing this as his mathematical lifelong quest. I read in one of these emails: “3D fluids and turbulence (as in the real world) is an area that has been puzzling me since the 90-91 because it seems its natural setting or comfort zone is not really identified yet. I am thinking, focusing really on *What is a manifold*”. In another mail, I read: “Riemann introduced the notion of manifold, Gelfand, then Grothendieck and then Connes recast the geometrical aspect in terms of the algebra and the quality of functions. The two viewpoints are in duality.”

A few months ago, he sent me to read a 1992 paper by Hikosaburo Komatsu on hyperfunctions and microfunctions, with the comments: “Here is a marvelous example of a transfer of information explaining the history of ideas. [...] I liked so much how the paper first explained the ease of the 1-dimensinal case (in terms of what we know from before 1900), and then the modern story with the competitions between wave front set perspective of one school and the sheaf/algebra perspective of another. [...] I think this sheaf of hyperfunctions is a key point for me in a three decade hiatus to define smooth structures in real dimension one that will accommodate the vastly successful theory of dynamics in one dimension which provides the tool and the language to be able to ‘explain’ some of the missing parts of the numerical universalities discovered numerically by physicists in the 70’s. The non-missing part achieved by the end of the 80’s only worked for integer critical points using Teichmüller theory, even though the numerical theorems were known for all critical exponents.” In another email, again, on the same topic: “The thread with which I am preoccupied is the discussion of space where physical reality seems to take place which, by the way, can be reformulated with charts in terms of the notion of function or with algebra in terms of ideals in an algebra of functions”.

We are back to the most fundamental questions: What is a manifold? What is a function? What is space?

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3. Gopal Prasad Professorship Established at IAS

The Institute of Advanced studies, Princeton, USA (IAS) proudly announced on 28th April, 2022, the creation of the Gopal Prasad Professorship in recognition of prolific mathematician and six-time Member of IAS, Gopal Prasad. The professorship, endowed with a gift from the Prasad family, ensures that future generations of scholars, from all regions of the world, have the opportunity to benefit from the unique environment of discovery at IAS.

Peter Sarnak, current Professor in the School of Mathematics, has been selected as the inaugural Gopal Prasad Professor. The professorship is to be held by Faculty in the Schools of Mathematics and Natural Sciences.

"For half a century the Prasad family has been intimately associated with the work of the IAS," stated David Nirenberg, IAS Director and Leon Levy Professor. "It is therefore a special joy to see the Prasad name permanently associated with the Institute and its enduring mission. It feels especially fitting for this professorship to be shared by the Schools of Mathematics and Natural Sciences, given that close collaborations between these Schools have produced so many fundamental insights and discoveries."

Gopal Prasad, considered a leading expert on Lie groups and algebraic groups, is currently the Raoul Bott Collegiate Professor Emeritus of Mathematics at the University of Michigan. He joined IAS as a Member for the first time in 1973, returning every decade through 2013. Over the years-not only those spent at IAS-he enjoyed fruitful collaborations and discussions with Faculty including Harish-Chandra, Armand Borel, Robert Langlands, Pierre Deligne, and Peter Sarnak.

"Like so many scholars around the world, my father is a direct beneficiary of the creative and collaborative environment at IAS, which richly informed his research and academic partnerships," explained Anoop Prasad. "In establishing this professorship, my family is investing in the ongoing IAS legacy as an enabler and diffuser of scientific knowledge globally."

As a child, Anoop came to IAS with his parents in 1973 and spent a year at Crossroads Nursery School. During Gopal's subsequent visits to IAS, Anoop attended Johnson Park Elementary School and Princeton High School. After earning a Ph.D. in theoretical physics from Caltech in 1997, Anoop joined D. E. Shaw where he is now a Managing Director and Head of the Equities Group. He joined the Friends of IAS in 2012 and has served on the Friends Executive Committee since 2018.

Ila Fiete, Gopal's daughter, also enjoyed her stays on the IAS campus and attended Princeton schools in her childhood. She is now a physicist and computational neuroscientist, currently serving as Professor in the Department of Brain and Cognitive Sciences within the McGovern Institute for Brain Research at the Massachusetts Institute of Technology.

The family's connection to IAS and intellectual progress also extends to Gopal's brothers: Pawan Kumar, Visiting Professor (1996-2002) in the School of Natural Sciences; Shrawan Kumar, Member (1988-89) in the School of Mathematics; and Dipendra Prasad, Member (2006-07 & 1992-93) in the School of Mathematics.

"I am very honored to be the inaugural Gopal Prasad Professor," remarked Sarnak. "Gopal's foundational works in Group theory and in the related areas of Geometry, Representation Theory, and Number Theory enjoy a broad and far-reaching impact. I, and many others, are regular users of his many theorems, as well as his masterful and insightful presentations thereof. His long association with the Institute, and with my colleagues, makes this Professorship special for me."

Peter Sarnak has made major contributions to number theory and to questions in analysis motivated by number theory. His research in mathematics is wide-ranging, focusing on the theory of zeta functions and automorphic forms with applications to number theory, combinatorics, and mathematical physics.

Gopal Prasad received his Ph.D. in Mathematics from the University of Bombay, India, in 1976. In 1975, he began an appointment at the Tata Institute of Fundamental Research, and was named a Professor in 1984. He moved to the United States in 1991, was appointed a Professor of Mathematics at the University of Michigan in 1992, and in 2008, he was named the Raoul Bott Collegiate Professor of Mathematics. His numerous visiting appointments have included stays at

Yale University and the Mathematical Sciences Research Institute, Berkeley.

"My visits to the Institute were very inspiring and fruitful," recalled Gopal Prasad. "I found the atmosphere there to be extremely conducive to research and collaboration, unencumbered by any responsibilities and I learnt much from discussions with the Faculty and Members."

On May 26, 2022, IAS celebrated the establishment of the Gopal Prasad Professorship, The event began with opening remarks from IAS Director and Leon Levy Professor David Nirenberg, who noted the Prasad family's longtime association with the Institute, both as Members and Friends, and Professor James Stone, who extended his thanks for the Prasad family's generous gift to the School of Mathematics and Natural Sciences, which helps ensure future scholars have the opportunity to benefit from the uniquely edifying environment at IAS.

It was followed by a lecture delivered by inaugural Gopal Prasad Professor Peter Sarnak and Robert and Luisa Fernholz Professor Akshay Venkatesh. They provided an overview of the impact Gopal Prasad's work has had on mathematics, specifically mass formulae, geometries and dynamics. The event continued with lectures from past Member Andrei Rapinchuk (2005, 2018) and past Member Brian Conra (1997-98) on Gopal Prasad's contribution to the arithmetic theory of algebraic groups and matrix group dynamics, respectively.

From left to right: Brian Conrad, Stanford Uni.; Shrawan Kumar, Uni. Of North Carolina; Gopal Prasad; Chris Skinner, Princeton.

Gopal Prasad; George Losztig, Shrawan Kumar

Gopal Prasad; Pierre Deligne, IAS

The event concluded with a dinner celebrating Gopal Prasad and his family - where Ila and Anoop, Gopal's children, and his brother and past Member (1988-89) Shrawan Kumar spoke about their family and IAS, in addition to remarks from Nirenberg, Sarnak, and Tasho Kaletha, past Veblen Research Instructor (2010-13) and von Neumann Fellow (2020).

Gopal Prasad's Response:

Thank you, Peter. I am touched and honored beyond words. It has been a joy to hear the mathematical talks this afternoon and to celebrate together this evening. I would like to thank Peter Sarnak, Akshay Venkatesh, Andrei Rapinchuk, Brian Conrad, and Tasho Kaletha for their beautiful exposition and kind words. I would also like to thank George Lusztig for sharing his wisdom and friendship. I have greatly admired Peter Sarnak for his wide-ranging scholarship and insightful work and for his easy accessibility to discuss mathematics with both senior and junior members of the mathematical community. In addition to his own sweeping research, Peter contributes to the joint effort of mathematics through his generous enthusiasm and appreciation for the work of others. It is a great honor for me that he has the professorship bearing my name. We are each called to do what little we can as individuals in the edifice of human endeavors. If we are extraordinarily fortunate, our individual efforts are enriched and lifted by others' cooperation and collaboration, and together as humans great things become possible. The IAS represents this kind of achievement of human excellence. It is humbling to be intertwined with this institution even in a small way.

In my case, the cooperation and collaboration began with many influences. From literally walking barefoot on hot dusty roads to our impoverished school where there were nevertheless a few highly dedicated mathematics teachers. I did not know there were careers in Mathematical research, but my naivete, the bravado of youth, and an intuition not justified by evidence at that point in my life that Mathematics might be a vast uncharted planet on which I had visited but a few small islands, led me to leave our family business for further study. While at Patna University, I corresponded broadly, receiving a letter from Bertrand Russell, Linus Pauling, and most importantly, an IAS prospectus, all of which opened the curtain to the world beyond. My research work began at TIFR, where M. S. Raghunathan's role as my advisor was immense. A visit to Yale in 1972 at the invitation of Dan Mostow changed my life, both because it signaled that I might be able to make my own mathematical contributions whilst allowing me to provide for my young family. On the first of many visits to IAS as a member in 1973, at the invitation of Armand Borel, I was influenced by the Mathematical activity around me and the conversations with many visiting and permanent members including over the years, Harishchandra, Pierre Deligne, and Bob Langlands, who shaped my work. The unique atmosphere at IAS – unencumbered by formal duties – allowed for long-term thinking and productivity.

I'd like to express my great personal pride in the mathematical and scientific endeavors of my three younger brothers: Shrawan, Pawan, and Dipendra Prasad, all of whom have also been IAS members, and two of whom are here today. I thank both Dipendra and my youngest sister Kiran for traveling all the way from India for this function. Finally, the most sustained, lifelong collaboration has been with my wife, Indu, who has been an immeasurable support in all my endeavors. She has given me two extremely talented, kind children, and helped to raise them with the values they possess. I am grateful that they strive to help us and contribute to society in every way they are able, including their endowment to my beloved Institute, which I know will nurture the future collaboration and community that I am grateful to have experienced here.

About the Institute

The Institute for Advanced Study has served the world as one of the leading independent centers for theoretical research and intellectual inquiry since its establishment in 1930, advancing the frontiers of knowledge across the sciences and humanities. From the work of founding IAS faculty such as Albert Einstein and John von Neumann to that of the foremost thinkers of the present, the IAS is dedicated to enabling curiosity-driven exploration and fundamental discovery. Each year, the Institute welcomes more than 200 of the world's most promising post-doctoral researchers and scholars who are selected and mentored by a permanent Faculty, each of whom are preeminent leaders in their fields. Among present and past Faculty and Members there have been 35 Nobel Laureates, 42 of the 60 Fields Medalists, and 22 of the 25 Abel Prize Laureates, as well as many MacArthur Fellows and Wolf Prize winners.

Sources:

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4. What is happening in the Mathematical world?

Devbhadra V. Shah
Department of Mathematics, VNSGU, Surat; Email: drdvshah@yahoo.com

4.1 A breakthrough towards the Chowla conjecture


Maksym Radziwiłł Harald Helfgott

In the work published last March, *Harald Helfgott* (Right) of the University of Göttingen in Germany and *Maksym Radziwiłł* (Left) of the California Institute of Technology presented an improved question about the relationships between integers. They studied random walks on expander graphs in order to prove a strong statement about the prime factorization of consecutive integers, which in particular constitutes major progress towards a conjecture of S. D. Chowla.

The Chowla conjecture predicts that whether one integer has an even or odd number of prime factors does not influence whether the next or previous integer also has an even or odd number of prime factors.

That seemingly straight forward inquiry is entangled with some deep unsolved questions about the primes. It was only a few years ago that, progress was made on the question, when *Terence Tao* of the University of California, Los Angeles proved an easier version of the conjecture called the *logarithmic Chowla conjecture*. But while the technique he used was found to be innovative and exciting, It did not seem to be of much help in making headway on related problems, including ones about the primes.

Now Helfgott and Radziwiłł have introduced a new way. Their solution, which pushes techniques from graph theory squarely into the heart of number theory, has reignited hope that the Chowla conjecture will deliver on its promise.

Chawla's conjecture is understood in terms of the Liouville function, which assigns to integers a value of -1 if they have an odd number of prime factors and $+1$ if they have an even number. The conjecture predicts that there would be no correlation between the values that the Liouville function takes for consecutive numbers.

In 2015, *Radziwiłł* and *Kaisa Matomäki* of the University of Turku in Finland studied the behavior of the Liouville function over short intervals. It wasknown that, on an average, the function is $+1$ half the time and -1 half the time. But it was still possible that its values might cluster, cropping up in long concentrations of either all $+1$s or all -1s. Matomäki and Radziwiłł proved that such clusters almost never occur. Their work, published the following year, established that if you choose a random number and look at its hundred or thousand nearest neighbors, roughly half have an even number of prime factors and half an odd number. Tao came in within months, he saw a way to build on Matomäki and Radziwiłł's work to attack a version of the problem that is easier to study, the logarithmic Chowla conjecture.

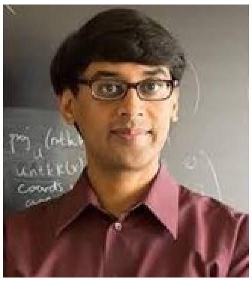
Tao's work was a complete breakthrough, but it would not lead to the natural next steps in the direction of problems more like the twin primes conjecture. Five years later, Helfgott and Radziwiłł managed to do what Tao could not - they could uphold the same conclusion over samples involving far fewer number of neighbors. The parity of the number of prime factors of an integer is thus not correlated with that of its neighbors.

Source: *https://www.quantamagazine.org/mathematicians-outwit-hidden- number- conspiracy - 20220103/*

4.2 Manjul Bhargava proves van der Waerden's Conjecture

Manjul Bhargava, Fields Medallist of 2014, has recently submitted a paper "Galois groups of random integer polynomials and van der Waerden's Conjecture" in Archive on 12 November 2021 in which he proves van der Waerden's Conjecture.

Consider for a fixed $n \geq 1$, monic polynomials $f(x) = x^n + a_1x^{n-1} + \cdots + a_n$ with integer coefficients satisfying $max\{|a_1|, \ldots, |a_n|\} \leq H$. There are $(2H+1)^n$ such polynomials. If $\alpha_1, \alpha_2, \ldots, \alpha_n$ are the n roots of $f(x)$ in $\mathbb{C}$, then the field $K = \mathbb{Q}(\alpha_1, \alpha_2, \ldots, \alpha_n)$ is a Galois extension of $\mathbb{Q}$ and the Galois group of the extension $K/\mathbb{Q}$ is a subgroup of the symmetric group S_n. The question is how many of these $(2H+1)^n$ polynomials have the associated Galois group a proper subgroup of S_n? If we take polynomials with the constant term $a_n = 0$, then these $(2H+1)^{n-1}$ polynomials have the Galois group a subgroup of S_{n-1} hence a proper subgroup of S_n.



For $H \geq 2$, let $E_n(H)$ be the number of monic integer polynomials $f(x) = x^n + a_1x^{n-1} + \cdots + a_n$ of degree n with $|a_i| \leq H$, and whose Galois group is not the full Galois group S_n. Then by the above argument $E_n(H) >> H^{n-1}$, i.e. $O(H^{n-1})$ is a lower bound for $E_n(H)$. Classical reasoning due to Hilbert shows that $E_n(H) = o(H^n)$; this means in a sense that "almost all" monic integral polynomials of degree n are irreducible and have Galois Group S_n. This is also described by saying that (asymptotically) one hundred percent of monic integer polynomials have Galois group S_n. Van der Waerden's conjecture is a stronger version of this result of Hilbert.

In 1936, van der Waerden conjectured that $O(H^{n-1})$ should in fact also be the correct upper bound for the count of such polynomilas. The conjecture has been known earlier for degrees $n \leq 4$, due to the work of van der Waerden and Chow and Dietmann.

Improvements using varied techniques and ideas have appeared in the literature over the years. Prior to the paper of Bhargava, the best record was held by David Lowry-Duda and his collaborators when they showed (2022, Archive) that $E_n(H) = O(H^{n-\frac{2}{3}+\frac{2}{(3n+3)}+\epsilon})$. As a remarkable improvement, now Manjul Bhargava proves van der Waerden's Conjecture for all degrees n outright, by showing that $E_n(H) = O(H^{n-1})$ for every fixed n.

This is a wonderful achivement proving the 85 year old conjecture!

Sources:

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4.3 A major stride towards understanding century-old combinatorics problem

A new research shows how to create longer disordered strings than was thought possible, proving that a well-known recent conjecture is "spectacularly wrong".

The mathematician *Ben Green* of the University of Oxford has made a major stride toward understanding a nearly 100-year-old combinatorics problem, showing that a well-known recent conjecture is "not only wrong but spectacularly wrong". This work shows how to create much longer disordered strings of colored beads than was thought possible, extending a line of work from the 1940s that has found applications in many areas of computer science.

The conjecture, formulated about 17 years ago by *Ron Graham*, one of the leading discrete mathematicians of the past half-century, concerns how many red and blue beads you can string together without creating any long sequences of evenly spaced beads of a single color.

This problem is one of the oldest in Ramsey theory, which asks how large various mathematical objects can grow before pockets of order must emerge. The bead-stringing question is easy to state but deceptively difficult: For long strings there are just too many bead arrangements to try one by one.

It has been known for nearly a century that one cannot keep stringing beads indefinitely, without bringing in evenly spaced space. Once one choses the parameters for each color, one can string only so many beads before being forced to create an evenly spaced sequence that is longer than one is willing to tolerate. As the red and blue parameters increase, the overall number of beads one can string increases - but how quickly?

In a version of the problem in which one forbid even the shortest evenly spaced blue sequences, Graham speculated that a simple relationship holds: The length of the longest possible beadstring is roughly the square of the red-bead parameter. All the numerical data mathematicians have accumulated supports Graham's conjecture. But now Green has proved the conjecture wrong. He has shown how to create much longer bead strings than Graham predicted. Green's construction, which blends geometry and dynamical systems to fashion the disordered beadstrings, builds on an earlier bead-stringing construction that has found applications in subjects from matrix multiplication to cryptography. This kind of construction is very important for questions in computer science.

Graham died last year at the age of 84, seven months before Green presented his work.

Sources: *https://www.quantamagazine.org/oxford-mathematician-advances-century-old- combinatorics -problem-20211215/*

4.4 Centuries-old 'impossible' math problem cracked under quantum entanglement

In 1779, Leonhard Euler proposed a problem (called Euler's officer problem) as follows: You're commanding an army with six regiments. Each regiment contains six different officers of six different ranks. Can you arrange them in a 6-by-6 square without repeating a rank or regiment in any given row or column?

Euler couldn't find such an arrangement, and later computations proved that there was no solution. In fact, a paper published in 1960 used the newfound power of computers to show that 6 was the one number over 2 where no such arrangement existed.

As a new study posted to the preprint database arXiv finds that you can arrange six regiments of six officers of six different ranks in a grid without repeating any rank or regiment more than once in any row or column...if the officers are in a state of quantum entanglement. The paper, which has been submitted for peer review at the journal Physical Review Letters, takes advantage of the fact that Quantum objects are in an intermediate or indeterminate state until they are measured, and in one deterministic state after they are measured.

In Euler's classic problem, each officer has a static regiment and rank. But a quantum officer might occupy more than one regiment or rank at once. A single officer could be either a Red Regiment first lieutenant or a Blue Regiment captain or, theoretically, any other combination. Also, the officers on the grid can be in a state of quantum entanglement, i. e. Crucially, the quantum states that make up these officers are entangled, which involves the correlation between different entities. For example, if a Officer No. 1, a Red Regiment first lieutenant, is entangled with Officer No. 2, a blue Regiment captain, then if we observe that the Officer No. 1 is a Red Regiment first lieutenant, then immediately know that Officer No. 2 must be a blue Regiment captain, and vice versa. This is the key to finding solution to the Euler's problem.

Adam Burchardt

Using brute-force computer power, the authors of the new paper, led by Adam Burchardt, a postdoctoral researcher at Jagiellonian University in Poland, proved that filling the grid with quantum officers made a solution possible. Surprisingly, the entanglement has its own pattern; Officers are only entangled with officers of ranks one step below or above them, while regiments are also only entangled with adjacent regiments.

The result could have real impact on quantum data storage. Entangled states can be used in quantum computing to ensure that data is safe even in the case of an error - a process called quantum error correction.

Sources:

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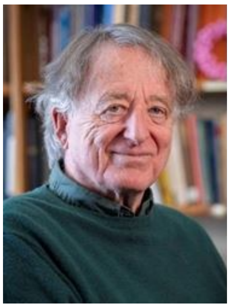
2. *https://www.livescience.com/math-puzzle-quantum-solution*

4.5 Awards

4.5.1 Dennis Parnell Sullivan, Uniter of Topology and Chaos, wins the 2022 Abel Prize



US mathematician *Dennis Sullivan* has won the prestigious Abel Prize in mathematics, for his contributions to topology - the study of qualitative properties of shapes - and related fields. His Majesty King Harald officially presented the Prize at a ceremony in Oslo, Norway, on May 24, 2022.

"Sullivan has repeatedly changed the landscape of topology by introducing new concepts, proving landmark theorems, answering old conjectures and formulating new problems that have driven the field forwards," says the citation for the 2022 Abel Prize, which was announced by the Norwegian Academy of Science and Letters, based in Oslo, on March 23, 2022. Throughout his career, Sullivan has moved from one area of mathematics to another and solved problems using a wide variety of tools, "like a true virtuoso", the citation added. The prize is worth 7.5 million Norwegian Kroner (about US $8,54,000). Details of his work and career is given in article 2 of this issue.

Source: *https://www.nature.com/articles/d41586-022-00841-w*

[This issue also contains a detailed article on the work of Dennis Sullivan, by Athanase Papadopoulos]

4.5.2 George Lusztig receives 2022 Wolf Prize for his work on representation theory

George Lusztig, the Abdun-Nur Professor of Mathematics at MIT, has been awarded the prestigious 2022 Wolf Prize in Mathematics for his work on geometric representation theory and algebraic groups.

The Israel-based Wolf Foundation cited the American-Romanian mathematician "for ground breaking contributions to representation theory and related areas." Lusztig is known for his work on representation theory, in particular for the objects closely related to algebraic groups, such as finite reductive groups, Hecke algebras, p-adic groups, quantum groups, and Weyl groups.

His contributions include the determination of the character table of finite reductive groups, development of canonical bases for quantum groups and Hecke algebras, and introduction of the concept of total positivity.

"His work is characterized by a very high degree of originality, an enormous breadth of subject matter, remarkable technical virtuosity, and great profundity in getting to the heart of the problems involved," says the Wolf Foundation. "Lusztig's ground breaking contributions mark him as one of the great mathematicians of our time."

Growing up in Romania, Lusztig discovered an early passion for mathematics. In eighth grade, he began representing Romania in the International Mathematical Olympiad, and was awarded silver medals in 1962 and 1963. After graduating from the University of Bucharest in 1968, he received his M.A. and Ph.D. from Princeton University in 1971 under *Michael Atiyah* and *William Browder*. He was a professor at the University of Warwick from 1974 to 1977, and joined the MIT mathematics faculty in 1978.

His work helped to lead modern representation theory, with fundamental new concepts that include the character sheaves, the "Deligne-Lusztig" varieties, and the "Kazhdan-Lusztig" polynomials. Lusztig's first breakthrough came with Pierre Deligne around 1975, with the construction of Deligne-Lusztig representations. He obtained a complete description of the irreducible representations of reductive groups over finite fields.

"Lusztig's description of the character table of a finite reductive group rates as one of the most extraordinary achievements of a single mathematician in the 20th century," says the Wolf Foundation. "To achieve his goal, he developed a panoply of techniques which are in use today by hundreds of mathematicians."

In 1979, *David Kazhdanand* Lusztig defined the "Kazhdan-Lusztig" basis of the Hecke algebra of a Coxeter group and stated the "Kazhdan-Lusztig" conjecture. The "Kazhdan-Lusztig" conjecture led directly to the "Beilinson-Bernstein" localization theorem, which, four decades later, remains our most powerful tool for understanding representations of reductive Lie algebras. Lusztig's work with MIT mathematics professor *David Vogan* then introduced a variant of the "Kazhdan-Lusztig" algorithm to produce "Lusztig-Vogan" polynomials. These polynomials are said to be fundamental to the understanding of real reductive groups and their unitary representations.

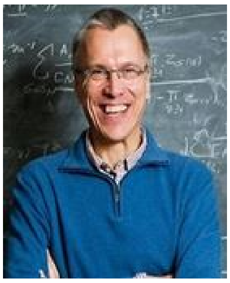
In the 1990s, Lusztig made seminal contributions to the theory of quantum groups. His contributions include the introduction of the canonical basis; the introduction of the Lusztig form; the quantum Frobenius and a small quantum group; and connections to the representation theory of affine Lie algebras. Lusztig's theory of the canonical basis has led to deep results in combinatorics and representation theory. Recent progress in representation theory and low- dimensional topology via "categorification" are connected to Lusztig's geometric categoryfication of quantum groups via perverse sheaves on quiver moduli.

He was appointed the Norbert Wiener Professor at MIT from 1999 to 2009, and currently holds the Abdun-Nur professorship. In 2014, Lusztig was the recipient of the 2014 Shaw Prize in mathematics.

The Wolf Prize in each field consists of a certificate and a monetary award of $1,00,000. Lusztig is among 345 scientists and artists to receive this honor since the prize was created in 1978 "for achievements in the interest of mankind and friendly relations amongst peoples" in fields ranging from physics, chemistry, mathematics, and agriculture to painting and sculpture, music, and architecture.

Source: *https://news.mit.edu/2022/mathematician-george-lusztig-receives-wolf-prize-0214*

4.5.3 Prof. Antti Kupiainen and his collaborators received George Pólya Prize in Mathematics



Antti Kupiainen

Remi Rhodes

Vincent Vargas

The Pólya Prize 2022 will be awarded to *Prof. Antti Kupiainen* (Left) from the University of Helsinki; and to his collaborators *Remi Rhodes* (Middle), *Vincent Vargas* (Right) (University of Geneva) for their work in the field of mathematical physics and probability theory.

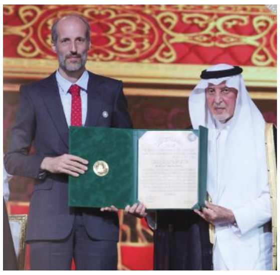
The committee cites that "The Pólya Prize is awarded to Kupiainen, Rhodes and Vargas for a rigorous justification of the DOZZ formula for three-point structure constants in Liouville Conformal Field Theory."

The George Pólya Prize in Mathematics is awarded by SIAM-Society for Industrial and Applied Mathematics, every four years for a significant contribution in an area of mathematics of interest to George Pólya not covered by the George Pólya Prize in Applied Combinatorics or the George Pólya Prize for Mathematical Exposition.

The prize was awarded at the 2022 SIAM Annual Meeting, held in a hybrid format on July 11-15, 2022 in Pittsburgh, Pennsylvania, United States.

Source: *https://www.miragenews.com/professor-antti-kupiainen-received-george-polya-716354/*

4.5.4 Prof. Martin Hairer and Prof. Nader Masmoudi recognized by 2022 King Faisal Prize



Prof. Martin Hairer

Two mathematicians were awarded jointly this year's King Faisal Prize for Science. The prizes were presented to them on March 29, 2022 in Riyadh, Saudi Arabia.

The first awardee *Prof. Martin Hairer*, is Chair in Probability and Stochastic Analysis at Imperial College's Mathematics Department. He has also been recipient of several coveted honours earlier, which include the Fermat prize in 2013, the Fröhlich prize and the Fields Medal in 2014, a knighthood in 2016, and the Breakthrough prize in Mathematics in 2020.

His work has been in the general area of probability theory with a focus on the analysis of stochastic partial differential equations. He recently developed the theory of regularity structures which gave a precise mathematical meaning to several equations that were previously outside the scope of mathematical analysis.

Prof. Nader Masmoudi

The other awardee, *Prof. Nader Masmoudi* is a Distinguished Professor of Mathematics at the New York University of Abu Dhabi and a head of his Research Center on Stability, Instability and Turbulence. He was able to unlock the mystery around many physics problems, which remained unsolved for centuries.

Dr. Masmoudi found a flaw in Euler's mathematical equations, which for more than 2 centuries described the motions of fluids under any circumstance. He discovered that these equations do not apply to all circumstances, as previously thought. His work helped solve many problems related to fluid-modeling like weather predictions.

Since 1979, King Faisal Prize has been awarded to 282 laureates from 44 different nationalities who have made distinguished contributions to serving Islam, Muslims, and humanity at large, and who have conducted outstanding research and made pivotal discoveries and findings in major fields of Science, Medicine, Islamic Studies, and Arabic Language & Literature.

Each prize category out of the 5 different categories is endowed with SR 7,50,000 (an equivalent of $2,00,000); a 24-carat gold medal, weighing 200 grams, and a Certificate written in Arabic calligraphy signed by the Chairman of the Prize board, Prince Khalid Al-Faisal, inscribed with the Laureate's name and a summary of their work which qualified them for the prize.

Source: *https://financialpost.com/globe-newswire/gene-editing-technologies-fluid-mechanics-bre -akthroughs -and-solutions-to-unfathomable-mathematical-equations-recognized-by-king-faisal-prize*

4.5.5 David Williamson receives 2022 AMS Steele Prize for seminal contribution

David Williamson, Chair of the Department of Information Science in the Cornell Ann S. Bowers College of Computing and Information Science, New York and professor of Operations Research and Information Engineering (ORIE) and his coauthor and Ph.D. advisor *Michel X. Goemans*, RSA Professor of Mathematics at the Massachusetts Institute of Technology will receive the 2022 American Mathematical Society (AMS) Steele Prize for seminal contribution to research for the paper "Improved Approximation Algorithms for Maximum Cut and Satisfiability Problems Using Semidefinite Programming".

The paper, published in 1995 in the Journal of the ACM, focuses on the Max-Cut problem - a core problem in combinatorial optimization - and has had a major, sustained impact on the fields of theoretical computer science and optimization theory. In their seminal work, Williamson and Goemans presented a new approximation algorithm for the Max-Cut problem. The algorithm introduced several key innovations that have become classic. Moreover, as time passed, their findings have become applicable to an even wider range of fields, with connections to complexity theory, cryptography, combinatorics, and algebra.

The Steele Prize for seminal contribution to Research is awarded for a paper that has proved to be of fundamental or lasting importance in its field, or a model of important research. The prize is awarded according to the following six-year rotation of subject areas: open, analysis/probability, algebra/ number theory, applied mathematics, geometry/topology, and discrete mathematics /logic. The Steele Prizes were established in 1970 in honor of *George David Birkhoff, William Fogg Osgood*, and *William Caspar Graustein*, and are endowed under the terms of a bequest from *Leroy P. Steele*.

Source: *https://cis.cornell.edu/david-williamson-receives-2022-steele-prize-american- mathematical -society*

4.5.6 Rafael Pass and Yanyi Liu Win NSA Best Cybersecurity Research Paper Competition

Rafael Pass Yanyi Liu

Rafael Pass, Professor of Computer Science at Cornell Tech, and coauthor Yanyi Liu, recently won the NSA's Best Cybersecurity Research Paper competition for their paper "On One-way Functions and Kolmogorov Complexity" which was presented at the 2020 IEEE Symposium on Foundations of Computer Science. The award is presented annually to researchers whose papers show "an outstanding contribution to cyber security science," with the goal of encouraging the development of the scientific foundations of cyber security.

The paper addresses an important question in the world of cryptography: Does an unbreakable code exist? In seeking answer to this question, One-way functions (OWFs) which were first proposed in 1976 by Whitfield Diffie and Martin Hellman, play a very important role. Formally, these functions are defined on strings of zeros and ones i.e. A function $f : \{0,1\}^* \to \{0,1\}^*$ is is one-way if f can be computed by a polynomial time algorithm, but any polynomial time randomized algorithm F that attempts to compute a pseudo-inverse for f succeeds with negligible probability. Hence, OWFs can be used to easily encode a message but it would be very hard (in the sense of complexity theory) for an unauthentic agents to decipher the coded message. One-way functions, in this sense, are fundamental tools for cryptography, personal identification, authentication, and other data security applications. While the existence of one-way functions in this sense is also an open conjecture, there are several candidates that have withstood decades of intense scrutiny. Some of them are essential ingredients of most telecommunications, e-commerce, and e-banking systems around the world.

The problem that concerns about, how hard it is to tell the difference between random strings of numbers and strings that contain some information, is known as Kolmogorov complexity. The theorem in the winning paper shows that if a certain version of Kolmogorov complexity is hard to compute, then true one-way functions do exist, and there is a clear-cut way to build one. Conversely, if this version of Kolmogorov complexity is easy to compute, then one-way functions cannot exist.

The finding suggests that instead of looking far and wide for candidate one-way functions, cryptographers could just concentrate their efforts on understanding Kolmogorov complexity. The proof is breakthrough work on the foundations of cryptography. The work has prompted cryptographers and complexity theorists to work together more closely, urging a burst of activity uniting their approaches.

Sources:

1. *https://tech.cornell.edu/news/rafael-pass-and-yanyi-liu-win-nsa-best-cybersecurity- research-paper-competition/*
2. *https://www.quantamagazine.org/researchers-identify-master-problem-underlying-all-cryptography-20220406/*

4.6 Obituary

4.6.1 Prof. Hale Trotter, 'pioneer and leader' in pure mathematics, passes away at the age of 91

Hale Freeman Trotter, an emeritus professor of mathematics, passed away at the age of 91on January 17, 2022 at Princeton, New Jersey.

He worked in various central areas of modern mathematics, and as a result his impact was very broad. He made significant contributions to group theory, knot theory and number theory. One of his outstanding accomplishments, the Trotter Product Formula, has had a major impact on mathematical physics and on functional analysis. The Johnson-Trotter Algorithm, another powerful and useful tool he developed, is a technique for generating complete lists of permutations. He also developed an interest in knot theory and in some of the calculational aspects of number theory, developing the Lang-Trotter conjecture through his joint work with Yale mathematician *Serge Lang*. He was a pioneer and leader of the use of the modern computer in pure mathematics. Trotter also wrote several textbooks on calculus in higher dimensions.

Born on May 30, 1931, in Kingston, Ontario, Trotter graduated with degrees in mathematics from Queen's University, getting his B.A. in 1952 and his M.A. in 1953. He then came to Princeton, where in the rich and exciting atmosphere, Trotter matured as a mathematician. After completing his Ph.D. in 1956, he began his career as the Fine Instructor for Mathematics at Princeton during 1956-58. He then taught at Queen's University from 1958-60 before returning to Princeton, where he stayed for the remainder of his career. He served as department chair during 1979-82 and associate director of Princeton University's Data Center during 1962-86. He transferred to emeritus status in 2000.

Sources:

1. *https://centraljersey.com/2022/02/08/hale-freeman-trotter-91/2*
2. *https://www.princeton.edu/news/2022/03/23/hale-trotter-pioneer-and-leader -pure-mathematic-dies-91*

4.6.2 Prof. Joel Moses, Professor Emeritus and computer science pioneer, passes away at 80

Institute Professor Emeritus *Joel Moses*, an innovative computer scientist and dedicated teacher who held multiple leadership positions at MIT, died on May 29, 2022 after a long battle with Alzheimer's and Parkinson's diseases at the age of 80.

Moses, a professor in the Department of Computer Science and Electrical Engineering and the former Engineering Systems Division, served as Associate department head, department head, dean of engineering, and provost during his distinguished career.

As a researcher, Moses is well-known for his work to develop *Macsyma* in the late 1960s, which was one of the first computer systems that could manipulate complex mathematical expressions, like those in algebra or calculus. The Macsyma program enables a computer to solve mathematical problems such as differentiating and integrating expressions, manipulating matrices, and deriving symbolic solutions of equations. Macsyma was faster and more accurate than other methods - problems in engineering or physics that would have taken six or seven months to calculate could now be solved in under an hour. The program influenced many powerful computational tools that are an outgrowth of Moses' research.

Moses was born in Palestine in 1941. From an early age, Moses showed an interest in mathematics When he was 13, his family emigrated from the new state of Israel to Brooklyn, New York. He attended Columbia University, and while his parents tried to convince him to become a doctor, Moses chose mathematics instead. As a master's student at Columbia, Moses first realized he could use computers to do maths. This laid the ground work for Macsyma and his later research. He earned a Ph.D. at MIT in 1967, working under the supervision of artificial intelligence pioneer *Marvin Minsky*, and wrote his thesis about the design and development of a computer program for performing symbolic integration.

After graduation, Moses joined the faculty as an assistant professor in computer science and began work on Macsyma in earnest two years later. An original member of the Artificial Intelligence Laboratory, his research efforts were centered on key algorithms that could simplify and integrate mathematical expressions.

Moses began his career as an administrator in 1974, when he was named associate director of the Laboratory for Computer Science and then associate head of the Department of Electrical Engineering and Computer Science (EECS) in 1978. As EECS department head in 1981, he launched a popular seminar series, affectionately called "the Moses Seminar," where faculty from every school gathered to talk about technical issues. During his time as provost, Moses led key budgeting initiatives and launched a retirement incentive plan that improved the financial footing of MIT while creating more openings for new faculty. After stepping down as provost in 1998, Moses rejoined the faculty and was named an Institute Professor one year later. He continued to be active in research and administration, most recently as acting director of MIT's engineering systems division.

Source: *https://news.mit.edu/2022/joel-moses-institute-professor-emeritus-dies-0531*

4.6.3 An eminent Mathematician and Ex Vice-Chancellor A. P. S. University, Rewa, Professor Satya Deo Tripathi passes away at the age of 78

Prof. Satya deo

A Mathematician of Repute and a person of par excellence is no more. He got a heart attack while sitting together with Professors V. P. Saxena and Ravi Kulkarni in the convention hall of Jawahar Lal Nehru University during an International conference, on 16th June 2022. Immediately he was rushed to Indian Spinal Injuries Centre in Delhi. On 19^{th} June 2022 at 9.30 am doctors declared him dead due to multiple organ failure. Prof. Satya Deo had kidney related issues and was on dialysis for about last 11 years. But these issues could not stop him in his academic pursuits and was academically active till the last day of consciousness.

Reacting to this sad incidence, Prof. Saxena said "this is the saddest moment for me as we were friends for more than thirty five years, meeting and talking frequently since he became Professor of Mathematics at R. D. University, Jabalpur. He was a thorough gentlemen, soft spoken and used to speak to the point. He had given several constructive suggestions during the coordination committee meetings chaired by the Honourable Governor of Madhya Pradesh in which we used to sit side by side".

Professor Satya Deo has been known for his outstanding research in Algebraic and Differential Topology, Topological and Differentiable Group Actions, Homology-Cohomology Theories, Cohomological Dimension Theory, Burnside Rings, Hopfian and Cohopfian Groups, and Spline Modules. After acquiring M.Sc. Degree in Mathematics from University of Allahabad in 1966, he did his Ph.D. in Algebraic Topology at University of Arkansas, Fayetteville, AR, in 1974, under supervision of Prof. John Keesee. He has to his credit over 60 research papers and a book entitled Algebraic Topology – a Primer, Hindustan Book Agency, New Delhi, 2003, TRIM Series #27. He had been an excellent teacher and he guided 17 students for Ph.D.

He started his teaching career in July 1966 and was a professor of Mathematics at R. D. University, Jabalpur from 1989 to 2006, Vice Chancellor, APS University, Rewa, MP from September 1998 to October 2001, and Visiting Professor, Harish-Chandra Research Institute, Allahabad, since July 2003 till his death.

Prof. Satya deo received several awards and honors including Fulbright Scholar at the University of Arkansas, USA, July 1971 - July 1973; Indo-US Fulbright Visiting Professorship Award to visit the University of Hawaii University of California, Berkeley (Summer 93); National Associate of the UGC, New Delhi for five years (1980-85); INSA-French Academy of Sciences Award to visit France, (Nov-Dec. 1985); Indo-German Cultural Exchange Award to visit West Germany (May-July 1987); Elected Fellow of the National Academy of Sciences, India (F.N.A.Sc.) in 1991; Elected Fellow of the International Academy of Physical Sciences, Allahabad 2002; Elected President, Indian Mathematical Society, One term, April 2000 to March 2001.

He was known for his wonderful organisational capacity and working tirelessly for the premier Indian organization like NASI and IMS. His stay and work in HRI, Allahabad has left a permanent impression on the institution. His association with The Mathematics consortium is unforgettable. He contributed to a number of joint programs of TMC with NASI and in fact some are still in the pipeline. He was a strong bridge between mentioned Indian agencies. The void created by his sad demise may not be fulfilled again. We, the family of TMS and other Mathematician friends, will miss his lively presence amongst us.

Source: *https://www.hri.res.in sdeo*

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5. International Calendar of Mathematics Events

Ramesh Kasilingam
Department of Mathematics, IITM, Chennai;
Email: rameshk@iitm.ac.in

August 2022

- August 8-19, 2022, *Elliptic curves and the special values of L-functions (HYBRID)*, Ramanujan Lecture Hall and online, ICTS, India.
- August 23-26, 2022, Workshop on Infinite Dimensional Analysis Buenos Aires, Argentina. *wid aba22.dm.uba.ar/*
- August 30 - September 2, 2022, SIAM Conference on Nonlinear Waves and Coherent Structures (NWCS22) University of Bremen, Bremen, Germany. *www.siam.or/conferences/cm/conference/nwcs22* on Zoom. math.as.uky.edu/oram20

September 2022

- September 2-5, 2022, Geometric applications of microlocal analysis Stanford University, Stanford, CA. *geometric-microlocal-2022.stanford.edu/*
- September 6-7, 2022, International Conference on Enumerative Combinatorics and Applications ICECA 2022, University of Haifa - Virtual. *ecajournal.haifa.ac.il/Conf/ICECA2022.html #about*

- September 8-9, 2022, *Heilbronn 2022 conference*, Bristol.
- September 11-16, 2022, Noncommutative harmonic analysis and quantum groups Bedlewo conference center, Bedlewo, Poland.
www.impan.pl/en/activities/banach-center/conferences/22-noncommutative
- September 12-16, 2022, Moduli spaces and geometric structures, Conference in honour of Oscar Garcia-Prada on the occasion of his 60^{th} birthday, ICMAT, Madrid, Spain.
www.icmat.es /congresos/2022/modOGP60/
- September 19-23, 2022 Probability and Analysis 2022, University of Wrocław and Wrocław University of Science and Technology, Wrocław, Poland. *panda.pwr.edu.pl/*
- September 12-16, 2022, *Elliptic curves and modular forms in arithmetic geometry*, celebrating Massimo Bertolini's 60^{th} birthday, Università de gliStudi di Milano, Via Festa del Perdono, 2022 Milano, Italia.
- September 28-29, 2022, The Second Conference on Mathematics and Applications of Mathematics (2^{nd} CMAM 2022), LMAM Laboratory, Mohamed Seddik Ben Yahia University, Jijel, Algeria. *cmam2022.mystrikingly.com/*
- September 29-October 1, 2022, ICGAMS 2022, Pimpri Chinchwad College of Engineering, Pune, India, *https://fe.pccoepune.com/Icgams2k22/Register.php*

October 2022

- October 4-7, 2022, MMSC 2022 - Workshop on Mathematical Modeling and Scientific Computing: Focus on complex processes and systems, Munich, Germany.
https://easychair.org/cfp/MMSC-2022
- October 15-16, 2022, 2^{nd} International Conference on "Orthogonal Polynomials, Special Functions and Computer Algebra: Applications in Engineering" Anand International College of Engineering, Jaipur, India. *anandice.ac.in/opsfca2022/index.html*
- October 15-16, 2022, *Québec-Maine Number Theory Conference 2022*, Université Laval, Québec, Canada.
- October 20-22, 2022, International E-Conference on Mathematical and statistical sciences: A Selçuk meeting, Selçuk University, Konya, Turkey. (Online). *icomss22.selcuk.edu.tr/*
- October 21-23, 2022, GROW 2022: Graduate Research Opportunities for Women Duke University, Durham, NC. *sites.duke.edu/grow2022/*
- October 22, 2022, UNYTS 2022: 3^{rd} Upstate New York Topology Seminar Syracuse University, Syracuse, NY USA. *https://clamille.github.io/UNYTS2022.html*
- October 27-30, 2022, The 5^{th} Mediterranean International Conference of Pure & Applied Mathematics and Related Areas (MICOPAM 2022), Sherwood Exclusive Lara Hotel (Ultra All-Inclusive) in Antalya, Turkey. *micopam.com*

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6. Problem Corner

Udayan Prajapati
Head, Mathematics Department, St. Xavier's College, Ahmedabad
Email: udayan.prajapati@gmail.com

In the January 2022 issue of TMC Bulletin, we posed a problem from Algebra for our readers. We have received two solutions for that problem. One solution is unclear in some aspects, so its correctness is doubtful. We will be presenting the other solution in this section sent by an anonymous reader.

We also pose a problem on Diophantine equation, for our readers. Readers are invited to email their solutions to Udayan Prajapati (udayan64@yahoo.com), Coordinator, Problem Corner, on or before 10th September, 2022. Most innovative solution will be published in the subsequent issue of the bulletin.

Problem: Find all real solutions to the following system of equations:

$$(x-1)(y^2+6) = y(x^2+1) - - - - - - (1)$$
$$(y-1)(x^2+6) = x(y^2+1) - - - - - - (2)$$

[Posed in TMC Bulletin, Vol-3, Issue-3, Jan 2022, by Dr. Vinay Acharya, Fergusson college, Pune.]

Solution: We write the given equations as quadratic in x:

$$yx^2 - (y^2+6x)x + (y^2+y+6) = 0;$$
$$(y-1)x^2 - (y^2+1)x + 6(y-1) = 0$$

The condition for these two equations have a common root is
$[(y^2+y+6)(y-1) - 6y(y-1)]^2 = [-6(y^2+6)(y-1) + (y^2+1)(y^2+y+6)] \times [-y(y^2+1) + (y-1)(y^2+6)]$,
or $(y-1)^2(y^2-5y+6)^2 = [y^4-5y^3+13y^2-35y+42] \times [-y^2+5y-6]$,
or $(y^2-5y+6)[(y-1)^2(y^2-5y+6) + (y^4-5y^3+13y^2-35y+42)] = 0$,
or $(y^2-5y+6)[(y^2-2y+1)(y^2-5y+6) + (y^2-5y+6)(y^2+7)] = 0$
or $(y-2)^2(y-3)^2.2(y^2-y+4) = 0$.
Therefore, since $y^2-y+4=0$ has non-real roots, we get $y=2$ or $y=3$.
For $y=2$, (1) becomes $x^2-5x+6=0$ so that $x=2$ or 3.
Similarly, for $y=3$, (1) gives $x=2$ or 3.
Thus, if (x,y) is a solution of (1) and (2), then $(x,y) \in \{(2,2),(2,3),(3,2),(3,3)\}$.
It can be seen that $(x,y) = (2,2),(2,3),(3,2)$ and $(3,3)$ are solutions of the given systems.
Hence, these are the only real solutions of the given system.

Problem for this issue

Find all the integral solutions to the following Diophantine equation:

$$x^2(y+1) + y^2(x+1) = 0$$

□□□

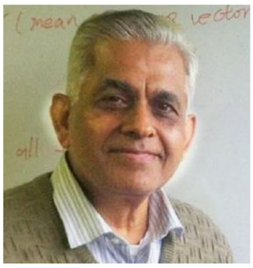
When a truth is necessary, the reason for it can be found by analysis, that is, by resolving it into simpler ideas and truths until the primary ones are reached. It is this way that in mathematics speculative theorems and practical canons are reduced by analysis to definitions, axioms and postulates. — Gottfried Leibniz

Source: *Quotes › Authors › G › Gottfried Leibniz › Mathematics*

7. TMC Activities

S. A. Katre, Lokmanya Tilak Chair, S. P. Pune University, Pune, Email: sakatre@gmail.com

7.1 An International Conference in Honour of Professor Ravi S. Kulkarni's 80th Birthday, May 21–25, 2022



An International Conference was organised by Bhaskaracharya Pratishthana (BP) in a hybrid mode at Pune in honour of Professor Ravi S. Kulkarni turning 80 this year. The main aim of the conference was to celebrate the influence of Ravi Kulkarni in Mathematics. Kulkarni's pioneering work from 1968 until 1998 falls in the broad area of differential geometry. From 2000 onwards, Kulkarni has influenced some unexpected directions in Group theory with ideas coming from geometry. Kulkarni has also been interested in general mathematics, mathematical philosophy, and Indian knowledge systems.

In the conference, there were talks by mathematicians from India and abroad, especially US, giving flavors from all these areas that intrigued Ravi over the years. Accordingly the conference was divided into three parts: *Differential geometry (May 21 & 24)*, Symmetries *from a geometric viewpoint (May 23)*, and May 25 was devoted to several lectures on general mathematics, mathematical philosophy and Indian mathematics. A Felicitation programme was held on Ravi Kulkarni's Birthday (May 22) from 4.30 p.m. to 7 p.m. (IST) presided by the Eminent Computer Scientist Padmabhushan Dr Vijay Bhatkar. Professors Karmeshu, Vinod Saxena, Satya Deo, Sudesh K. Khanduja, J. K. Verma, A. R. Shastri, Dilip Patil, S. A. Katre and various mathematicians from abroad expressed their felicitations to Prof. Kulkarni on this occasion.

The conference speakers included many friends, colleagues and former students of Prof. Ravi Kulkarni, who had interacted with him on these topics over the years. The timetable, especially on May 21 & 24, was prepared keeping in mind the time difference of the overseas locations of the speakers. Conference talks are available at the YouTube chanell of Bhaskaracharya Pratishthana. Proceedings of the conference will be published.

Organizing Committee: **S. A. Katre (Convener)**, S. P. Pune Uni. & BP, Pune; **Krishnendu Gongopadhyay (Convener)**, IISER Mohali, Punjab; **V. M. Sholapurkar**, S. P. College & BP, Pune; **V. V. Acharya**, Fergusson College & BP, Pune and **Vikas Jadhav**, Nowrosjee Wadia College, Pune.

Speakers:

Kulkarni@80: Differential geometry:

• Ara Basmajian (CUNY) • Sagun Chanillo (Rutgers Uni., New Brunswick) • S. G. Dani (CEBS, Mumbai) • Allan Edmonds (Indiana) • William Goldman (Maryland) • Yunping Jiang (CUNY) • Alexander Mednykh (Novosibirsk) • Ulrich Pinkall (TU Berlin) • Nitin Nitsure (TIFR, Mumbai) • Jose Seade (UNAM, Mexico) • Dragomir Saric (CUNY) • S. -T. Yau (Harvard, Fields Medalist 1982) • Mukut Mani Tripathi

Kulkarni@80: Symmetries.

• Gurmeet K. Bakshi (Punjab University, Chandigarh) • Anupam K. Singh (IISER Pune) • Siddhartha Sarkar (IISER Bhopal) • Soham Pradhan (Haifa University, Israel) • Jagmohan Tanti (BBAU Lucknow) • Anthony Weaver (CUNY) • Devendra Tiwari (BP, Pune) • Vikas Jadhav (Wadia College, Pune)

Kulkarni@80: General Mathematics

• Satya Deo (HRI) • Jugal Verma (IIT Bombay) • Rohit Parikh (CUNY) • M. A. Sofi (Kashmir Uni.) • Bankteshwar Tiwari (BHU) • Sudesh K. Khanduja (IISER Mohali).

7.2 A Regional Programme of TMC jointly with Bhaskaracharya Pratishthana.

On the occasion of the Kulkarni@80 Conference, a Workshop was organized on Applications of Mathematics for Students and Teachers during 9.30 am - 3.30 pm on 22^{nd} May 2022. It was coordinated by Dr. Ajay Deshmukh of BP. Speakers and titles of their talks were:

• S. A. Katre (S. P. Pune Univ.) Bhasaracharya's methods in RSA Cryptography • A. R. Shastri (IIT, Bombay) Platonic solids • V. D. Pathak (M.S. Univ., Vadodara) Planning of Loop Type Electric Distribution System Layout • Krishnendu Gongopadhyay (IISER, Mohali) Symmetries from a geometric viewpoint • Sudhir Ghorpade (IIT, Bombay) Introduction to coding theory.

7.3 A NASI-TMC Trimester programme

The Mathematics Consortium had initiated a NASI-TMC Trimester programme on Triangle Groups, Belyi Uniformization and Modularity organised by Bhaskaracharya Pratishthana, Pune. The First Trimester (Sept-Dec. 2021) and the Second Trimester (Jan.-April 2022) are now complete. The 3^{rd} Trimester has started from 1st May 2022 and will be over by 31^{st} August 2022.

The talks in the first trimester were related to the modular group, Riemann surfaces, Elliptic functions, Monodromy, Triangle groups, Maps and coverings, Riemann Roch Theorem, Hyperbolic, planar and spherical tessellations, modular curves, Arithmetic Fuchsian Groups, the Klein quartic, Triangular modular curves, Belyi's theorem and Grothendieck dessin d'enfant, fundamental domains and Poincaré's polygon theorem, Three point branched cover of the projective line, Belyi maps, Farey symbols and finite index subgroups of the modular group, Grothendieck-Teichmuller Theory, low genus triangular modular curves, Hecke group, ABC implies Mordell, finite index subgroups of the modular group and modular forms.

In the second trimester there were talks on Tuesdays and Thursdays. The talks were on the topics: Elliptic curves, modular forms and FLT, Taylor-Wiles Method, Galois representations attached to elliptic curves and modular forms, Fermat-like equations, Modular curves, Eichler-Shimura relation, Hilbert modular forms, Uniformization of elliptic curves and Shimura curves, Monodromy of the Schwarzian Equation on Riemann surfaces, Abelian varieties, Noncongruence modular curves, Modularity over totally real fields, Unbounded denominator conjecture, Modularity over $\mathbb{C}$ implies modularity over $\mathbb{Q}$, Cyclic covering of the projective line and arithmetic groups, Modular method approach for Diophantine equations, Rational isogenies of prime degree, Modularity of elliptic curves over totally real fields, Modularity of rigid Galois representations, Darmon's programme, and Torsion points of elliptic curves.

For more information see: *https://sites.google.com/view/bms2021/home?authuser=0*
or *www.bprim.org*

□□□

Not that the propositions of geometry are only approximately true, but that they remain absolutely true in regard to that Euclidean space which has been so long regarded as being the physical space of our experience. — Arthur Cayley QUOTESTATS.COM

In retrospect all great ideas take on a certain simplicity and inevitability, partly because they shape the whole subsequent development of the subject. — Raoul Bott
https://libquotes.com/raoul-bott

Celebration of 80th birthday of renowned Indian Mathematician Prof. Ravindra S. Kulkarni at Bhaskaracharya Pratishthan, Pune

First Indian supercomputer developer,
Padmabhushan Dr. Vijay Bhatkar Felicitating Prof. Ravindra Kulkarni

Prof. V. M. Sholapurkar, Dr. Vijay P. Bhatkar,
Prof. Ravindra S. Kulkarni, Prof. S. A. Katre

Floral Welcome

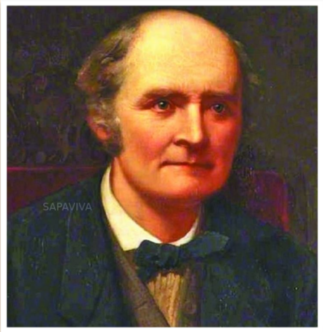
Offering gift

Gathering at BP

Prof. Kulkarni responding to the honor

Gottfried Withelm (von) Leibniz (1 July 1646 - 14 Nov. 1716)

A German Mathematician, Philosopher, Scientist and diplomat. Developed main ideas of differential and Integral calculus. Pioneer in the field of Mechanical calculator. One of the Pioneers in actuarial science. Devised Cramer's rule and Gaussian elimination method for solving systems of Linear equations. Developed theory of motion based on kinetic and potential energy.



Arthur Cayley (16 Aug. 1821 – 26 Jan. 1895)

A prolific British mathematician. Worked mostly on Algebra. First to define the concept of a group in the modern way. Postulated the Cayley-Hamilton theorem. Made fundamental contributions to algebraic geometry. Constructed the Chow variety of all curves in projective 3-space and founded algebra-geometric theory of ruled surface. Published a Treatise on Elliptic Functions.

Raoul Bott (24 Sep. 1923 - 20 Dec. 2005)

Hungarian-American Mathematician, Physicist, topologist. Proved Bott periodicity theorem and introduced Morse-Bott functions, Bott-Samelson varieties, the Bott-Chern classes, and the Bott residue formula for complex manifolds. Made contributions towards the index theorem and formulated related fixed-point theorems. Well known for the Borel-Bott-Weil theorem.

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43/16 Gunadhar Banglow, Erandawane, Pune 411004, India.
Email: tmathconsort@gmail.com Website: themathconsortium.in

Contact Persons

Prof. Vijay Pathak Prof. S. A. Katre
vdpmsu@gmail.com (9426324267) sakatre@gmail.com (9890001215)

Printers

AUM Copy Point, G-26, Saffron Complex, Nr. Maharana Pratap Chowk,
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